Near-Neighbor Search

Applications

Matrix Formulation

Minhashing

Example Application: Face Recognition

- We have a database of (say) 1 million face images.
- We want to find the most similar images in the database.
- Represent faces by (relatively) invariant values, e.g., ratio of nose width to eye width.

Face Recognition – (2)

- Each image represented by a large number (say 1000) of numerical features.
- Problem: given a face, find those in the DB that are close in at least ¾ (say) of the features.

Face Recognition – (3)

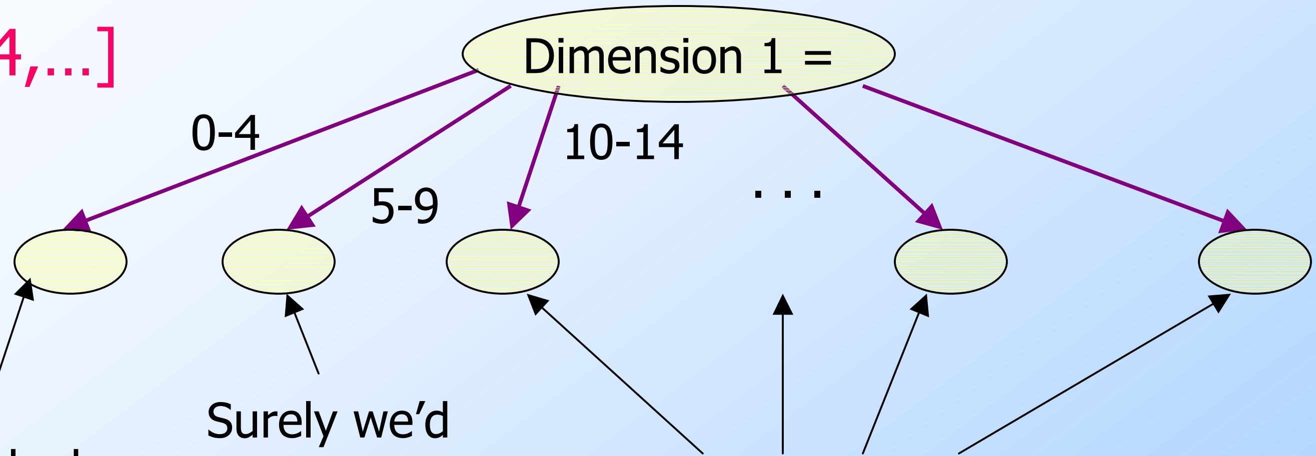
- *Many-one problem* : given a new face, see if it is close to any of the 1 million old faces.
- *Many-Many problem* : which pairs of the 1 million faces are similar.

Simple Solution

- Represent each face by a vector of 1000 values and score the comparisons.
- Sort-of OK for many-one problem.
- Out of the question for the many-many problem ($10^6 * 10^6 * 1000/2$ numerical comparisons).
- We can do better!

Multidimensional Indexes Don't Work

New face:
[6,14,…]



Maybe look here too, in case of a slight error.

Surely we'd better look here.

But the first dimension could be one of those that is not close. So we'd better look everywhere!

Another Problem: Entity Resolution

- Two sets of 1 million name-address-phone records.
- Some pairs, one from each set, represent the same person.
- Errors of many kinds:
  - Typos, missing middle initial, area-code changes, St./Street, Bob/Robert, etc., etc.

Entity Resolution – (2)

- Choose a scoring system for how close names are.
  - Deduct so much for edit distance > 0; so much for missing middle initial, etc.
- Similarly score differences in addresses, phone numbers.
- Sufficiently high total score -> records represent the same entity.

Simple Solution

- Compare each pair of records, one from each set.
- Score the pair.
- Call them the same if the score is sufficiently high.
- Unfeasible for 1 million records.
- We can do better!

Example: Similar Customers

- Common pattern: looking for sets with a relatively large intersection.
- Represent a customer, e.g., of Netflix, by the set of movies they rented.
- Similar customers have a relatively large fraction of their choices in common.

Example: Similar Products

- Dual view of product-customer relationship.
- Products are similar if they are bought by many of the same customers.
- E.g., movies of the same genre are typically rented by similar sets of Netflix customers.
  - Tricky: Sony and Samsung TV's are "similar," but not typically bought by the same customers.

Yet Another Problem: Finding Similar Documents

- Given a body of documents, e.g., the Web, find pairs of docs that have a lot of text in common, e.g.:
  - Mirror sites, or approximate mirrors.
  - Plagiarism, including large quotations.
  - Repetitions of news articles at news sites.

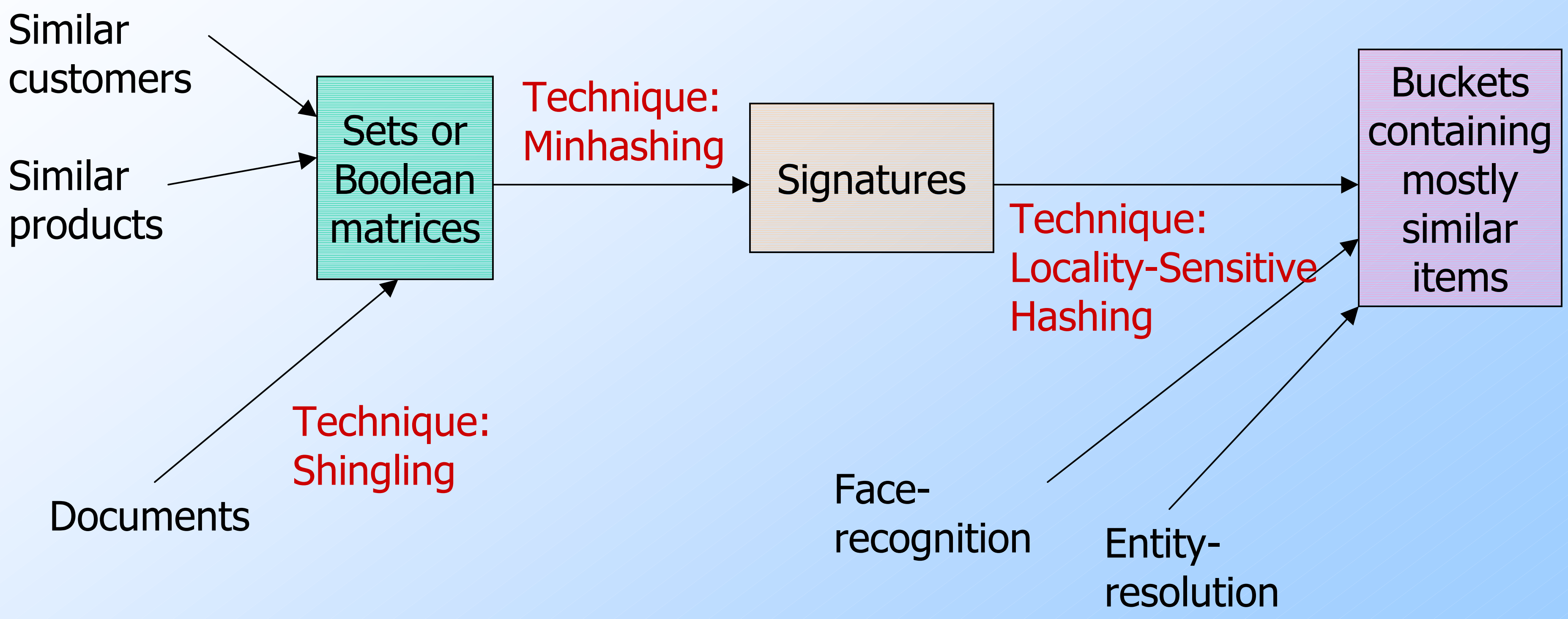
Complexity of Document Similarity

- For the face problem, there is a way to represent a big image by a (relatively) small data-set.
- Entity records represent themselves.
- How do you represent a document so it is easy to compare with others?

Complexity – (2)

- Special cases are easy, e.g., identical documents, or one document contained verbatim in another.
- General case, where many small pieces of one doc appear out of order in another, is very hard.

Roadmap


Similar customers
Similar products
Documents
Technique: Shingling
Sets or Boolean matrices
Technique: Minhashing
Signatures
Technique: Locality-Sensitive Hashing
Buckets containing mostly similar items
Face-recognition
Entity-resolution

Representing Documents for Similarity Search

1. Represent doc by its set of *shingles* (or *k* -grams).
2. Summarize shingle set by a *signature* = small data-set with the property:
   - Similar documents are very likely to have "similar" signatures.

- At that point, doc problem becomes finding similar sets.

Shingles

- A *k*-shingle (or *k*-gram) for a document is a sequence of *k* characters that appears in the document.
- Example: k=2; doc = abcab. Set of 2-shingles = {ab, bc, ca}.
  - Option: regard shingles as a bag, and count ab twice.

Shingles: Compression Option

- To compress long shingles, we can hash them to (say) 4 bytes.
- Represent a doc by the set of hash values of its *k*-shingles.
- Two documents could (rarely) appear to have shingles in common, when in fact only the hash-values were shared.

MinHashing

Data as Sparse Matrices

Jaccard Similarity Measure

Constructing Signatures

Basic Data Model: Sets

- Many similarity problems can be couched as finding subsets of some universal set that have large intersection.
- Examples include:
  1. Documents represented by their set of shingles (or hashes of those shingles).
  2. Similar customers or products.

From Sets to Boolean Matrices

- Rows = elements of the universal set.
- Columns = sets.
- 1 in the row for element *e* and the column for set *S* iff *e* is a member of *S*.

In Matrix Form

	S	T	U	V	W
a	1	1	0	1	0
b	1	0	1	1	0
c	1	0	0	1	0
d	0	1	0	0	1
e	1	0	1	0	1
f	1	1	0	1	1
g	0	1	0	1	1
h	0	1	0	1	0

S = {a,b,c,e,f}   T = {a,d,f,g,h}   U = {b,e}
V = {a,b,c,f,g,h}   W = {d,e,f,g}

Documents in Matrix Form

- Rows = shingles (or hashes of shingles).
- Columns = documents.
- 1 in row *r*, column *c* iff document *c* has shingle *r*.
- Expect the matrix to be sparse.

Aside

- We might not really represent the data by a boolean matrix.
- Sparse matrices are usually better represented by the list of places where there is a non-zero value.
  - E.g., movies rented by a customer, shingle-sets.
- But the matrix picture is conceptually useful.

Assumptions

1. Number of items allows a small amount of main-memory/item.
   - E.g., main memory =

     Number of items * 1000
2. Too many items to store anything in main-memory for each *pair* of items.

Similarity of Columns

- Remember: a column is the set of rows in which it has 1.
- The *similarity* of columns C_1 and C_2 = *Sim* (C_1,C_2) = is the ratio of the sizes of the intersection and union of C_1 and C_2.
  - *Sim* $(C_1,C_2) = |C_1 \cap C_2|/|C_1 \cup C_2|$ = *Jaccard similarity*.

Example: Jaccard Similarity

C_1	C_2		
0	1		*
1	0		*
1	1	*	*
0	0		
1	1	*	*
0	1		*

$$\text{Sim}(C_1, C_2) = 2/5 = 0.4$$

Outline: Finding Similar Columns

1. Compute signatures of columns = small summaries of columns.
   - Read from disk to main memory.
2. Examine signatures in main memory to find similar signatures.
   - Essential: similarities of signatures and columns are related.
3. Optional: check that columns with similar signatures are really similar.

Warnings

1. Comparing all pairs of signatures may take too much time, even if not too much space.
   - A job for Locality-Sensitive Hashing.
2. These methods can produce false negatives, and even false positives if the optional check is not made.

Signatures

- Key idea: "hash" each column *C* to a small *signature* *Sig* (C), such that:
  1. *Sig* (C) is small enough that we can fit a signature in main memory for each column.
  2. *Sim* (C_1, C_2) is the same as the "similarity" of *Sig* (C_1) and *Sig* (C_2).

An Idea That Doesn't Work

- Pick 100 rows at random, and let the signature of column *C* be the 100 bits of *C* in those rows.
- Because the matrix is sparse, many columns would have 00. . .0 as a signature, yet be very dissimilar because their 1's are in different rows.

Four Types of Rows

- Given columns C_1 and C_2, rows may be classified as:

	C_1	C_2
a	1	1
b	1	0
c	0	1
d	0	0

- Also, a = # rows of type a, etc.
- Note $Sim(C_1, C_2) = a/(a+b+c)$.

Minhashing

- Imagine the rows permuted randomly.
- Define "hash" function $h(C)$ = the number of the first (in the permuted order) row in which column C has 1.
- Use several (100?) independent hash functions to create a signature.

Minhashing Example

1	4	3
3	2	4
7	1	7
6	3	6
2	6	1
5	7	2
4	5	5

Input matrix

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Surprising Property

- The probability (over all permutations of the rows) that $h(C_1) = h(C_2)$ is the same as $Sim(C_1, C_2)$.
- Both are $a/(a+b+c)$!
- Why?
  - Look down columns C_1 and C_2 until we see a 1.
  - If it's a type-a row, then $h(C_1) = h(C_2)$. If a type-b or type-c row, then not.

Similarity for Signatures

- The *similarity of signatures* is the fraction of the rows in which they agree.
  - Remember, each row corresponds to a permutation or "hash function."

Min Hashing – Example

1	4	3
3	2	4
7	1	7
6	3	6
2	6	1
5	7	2
4	5	5

Input matrix

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.67	1.00	0	0

Minhash Signatures

- Pick (say) 100 random permutations of the rows.
- Think of *Sig* (C) as a column vector.
- Let *Sig* (C)[i] = according to the *i* th permutation, the number of the first row that has a 1 in column *C*.

Implementation – (1)

- Suppose 1 billion rows.
- Hard to pick a random permutation from 1…billion.
- Representing a random permutation requires 1 billion entries.
- Accessing rows in permuted order leads to thrashing.

Implementation – (2)

- A good approximation to permuting rows: pick (say) 100 hash functions.
- For each column c and each hash function h_i, keep a "slot" $M(i, c)$ for that minhash value.

Implementation – (3)

```
for each row r
    for each column c
        if c has 1 in row r
            for each hash function h_i do
                if h_i(r) is a smaller value than
                  M(i, c) then
                    M(i, c) := h_i(r);
```

Example

Row	C1	C2
1	1	0
2	0	1
3	1	1
4	1	0
5	0	1

$h(x) = x \bmod 5$
$g(x) = 2x+1 \bmod 5$

	Sig1	Sig2
$h(1) = 1$	1	-
$g(1) = 3$	3	-
$h(2) = 2$	1	2
$g(2) = 0$	3	0
$h(3) = 3$	1	2
$g(3) = 2$	2	0
$h(4) = 4$	1	2
$g(4) = 4$	2	0
$h(5) = 0$	1	0
$g(5) = 1$	2	0

Implementation – (4)

- If data is stored row-by-row, then only one pass is needed.
- If data is stored column-by-column
  - E.g., data is a sequence of documents

  represent it by (row-column) pairs and sort once by row.
  - Saves cost of computing $h(r)$ many times.