Near-Neighbor Search

Applications

Matrix Formulation

Minhashing

Example Problem --- Face Recognition

- We have a database of (say) 1 million face images.
- We are given a new image and want to find the most similar images in the database.
- Represent faces by (relatively) invariant values, e.g., ratio of nose width to eye width.

Face Recognition --- (2)

- Each image represented by a large number (say 1000) of numerical features.
- Problem: given the features of a new face, find those in the DB that are close in at least ¾ (say) of the features.

Face Recognition --- (3)

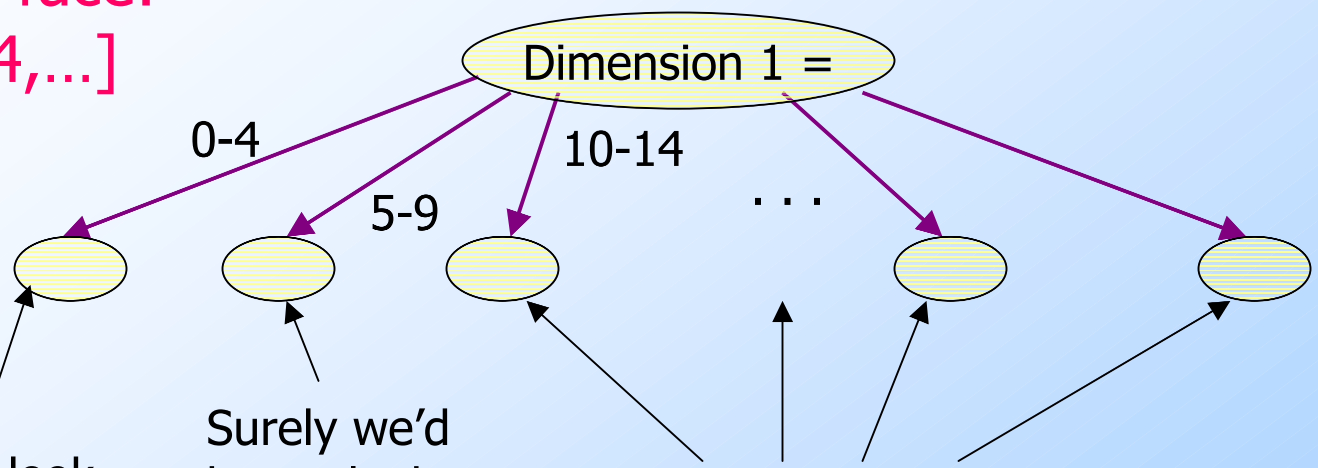
- *Many-one problem* : given a new face, see if it is close to any of the 1 million old faces.
- *Many-Many problem* : which pairs of the 1 million faces are similar.

Simple Solution

- Represent each face by a vector of 1000 values and score the comparisons.
- Sort-of OK for many-one problem.
- Out of the question for the many-many problem (10^6*10^6*1000 numerical comparisons).
- We can do better!

Multidimensional Indexes Don’t Work

New face:
[6,14,…]



Maybe look here too, in case of a slight error.

Surely we’d better look here.

But the first dimension could be one of those that is not close. So we’d better look everywhere!

Another Problem: Entity Resolution

- Two sets of 1 million name-address-phone records.
- Some pairs, one from each set, represent the same person.
- Errors of many kinds:
  - Typos, missing middle initial, area-code changes, St./Street, Bob/Robert, etc., etc.

Entity Resolution --- (2)

- Choose a scoring system for how close names are.
  - Deduct so much for edit distance > 0; so much for missing middle initial, etc.
- Similarly score differences in addresses, phone numbers.
- Sufficiently high total score -> records represent the same entity.

Simple Solution

- Compare each pair of records, one from each set.
- Score the pair.
- Call them the same if the score is sufficiently high.
- Unfeasible for 1 million records.
- We can do better!

Yet Another Problem: Finding Similar Documents

- Given a body of documents, e.g., the Web, find pairs of docs that have a lot of text in common.
- Find mirror sites, approximate mirrors, plagiarism, quotation of one document in another, "good" document with random spam, etc.

Complexity of Document Similarity

- The face problem had a way of representing a big image by a (relatively) small data-set.
- Entity records represent themselves.
- How do you represent a document so it is easy to compare with others?

Complexity --- (2)

- Special cases are easy, e.g., identical documents, or one document contained verbatim in another.
- General case, where many small pieces of one doc appear out of order in another, is very hard.

Representing Documents for Similarity Search

1. Represent doc by its set of *shingles* (or *k*-grams).
2. Summarize shingle set by a *signature* = small data-set with the property:
   - Similar documents are very likely to have "similar" signatures.

- At that point, doc problem resembles the previous two problems.

Shingles

- A *k*-shingle (or *k*-gram) for a document is a sequence of *k* characters that appears in the document.
- Example: k=2; doc = abcab. Set of 2-shingles = {ab, bc, ca}.
  - Option: regard shingles as a bag, and count ab twice.

Shingles: Aside

- Although we shall not discuss it, shingles are a powerful tool for characterizing the topic of documents.
  - $k = 5$ is the right number; $(\#\text{characters})^5$ >> # shingles in typical document.
- Example: "ng av" and "ouchd" are most common in sports articles.

Shingles: Compression Option

- To compress long shingles, we can hash them to (say) 4 bytes.
- Represent a doc by the set of hash values of its *k*-shingles.
- Two documents could (rarely) appear to have shingles in common, when in fact only the hash-values were shared.

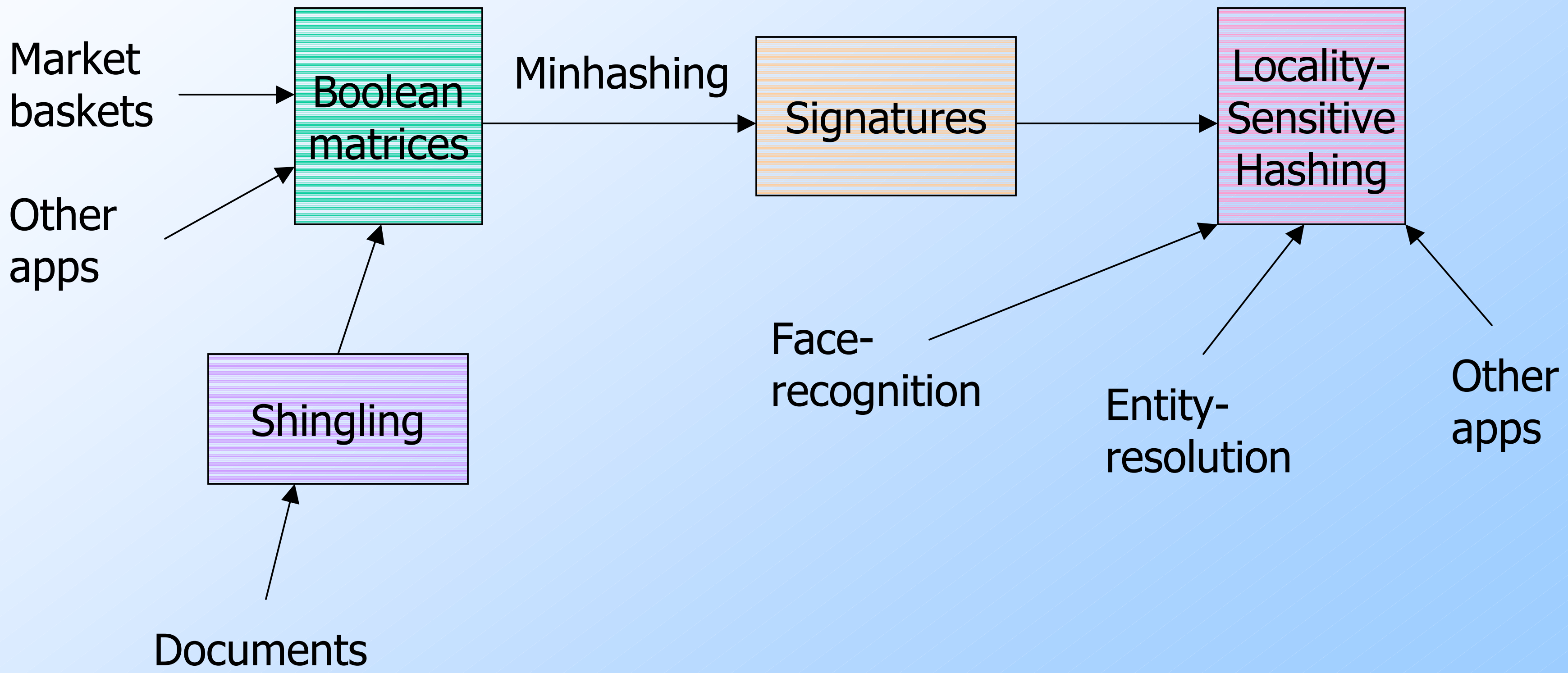
MinHashing

Data as Sparse Matrices

Jaccard Similarity Measure

Constructing Signatures

Roadmap


Market baskets
Other apps
Boolean matrices
Shingling
Documents
Minhashing
Signatures
Locality-Sensitive Hashing
Face-recognition
Entity-resolution
Other apps

Boolean Matrix Representation

- Data in the form of subsets of a universal set can be represented by a (typically sparse) matrix.
- Examples include:
  1. Documents represented by their set of shingles (or hashes of those shingles).
  2. Market baskets.

Matrix Representation of Item/Basket Data

- Columns = items.
- Rows = baskets.
- Entry (*r* , *c*) = 1 if item *c* is in basket *r* ; = 0 if not.
- Typically matrix is almost all 0's.

In Matrix Form

	m	c	p	b	j
{m,c,b}	1	1	0	1	0
{m,p,b}	1	0	1	1	0
{m,b}	1	0	0	1	0
{c,j}	0	1	0	0	1
{m,p,j}	1	0	1	0	1
{m,c,b,j}	1	1	0	1	1
{c,b,j}	0	1	0	1	1
{c,b}	0	1	0	1	0

Documents in Matrix Form

- Columns = documents.
- Rows = shingles (or hashes of shingles).
- 1 in row *r*, column *c* iff document *c* has shingle *r*.
- Again expect the matrix to be sparse.

Aside

- We might not really represent the data by a boolean matrix.
- Sparse matrices are usually better represented by the list of places where there is a non-zero value.
  - E.g., baskets, shingle-sets.
- But the matrix picture is conceptually useful.

Assumptions

1. Number of items allows a small amount of main-memory/item.
   - E.g., main memory =

     Number of items * 100
2. Too many items to store anything in main-memory for each *pair* of items.

Similarity of Columns

- Think of a column as the set of rows in which it has 1.
- The *similarity* of columns C_1 and C_2 = *Sim* (C_1,C_2) = is the ratio of the sizes of the intersection and union of C_1 and C_2.
  - *Sim* $(C_1,C_2) = |C_1 \cap C_2|/|C_1 \cup C_2|$ = *Jaccard measure*.

Example

C_1	C_2		
0	1		*
1	0		*
1	1	*	*
0	0		
1	1	*	*
0	1		*

$Sim(C_1, C_2) =$

$2/5 = 0.4$

Outline of Algorithm

1. Compute signatures of columns = small summaries of columns.
   - Read from disk to main memory.
2. Examine signatures in main memory to find similar signatures.
   - Essential: similarities of signatures and columns are related.
3. Optional: check that columns with similar signatures are really similar.

Warnings

1. Comparing all pairs of signatures may take too much time, even if not too much space.
   - A job for Locality-Sensitive Hashing.
2. These methods can produce false negatives, and even false positives if the optional check is not made.

Signatures

- Key idea: "hash" each column *C* to a small *signature* *Sig* (C), such that:
  1. *Sig* (C) is small enough that we can fit a signature in main memory for each column.
  2. *Sim* (C_1, C_2) is the same as the "similarity" of *Sig* (C_1) and *Sig* (C_2).

An Idea That Doesn't Work

- Pick 100 rows at random, and let the signature of column *C* be the 100 bits of *C* in those rows.
- Because the matrix is sparse, many columns would have 00. . .0 as a signature, yet be very dissimilar because their 1's are in different rows.

Four Types of Rows

- Given columns C_1 and C_2, rows may be classified as:

	C_1	C_2
a	1	1
b	1	0
c	0	1
d	0	0

- Also, *a* = # rows of type *a* , etc.
- Note $Sim(C_1, C_2) = a/(a+b+c)$.

Minhashing

- Imagine the rows permuted randomly.
- Define "hash" function $h(C)$ = the number of the first (in the permuted order) row in which column C has 1.
- Use several (100?) independent hash functions to create a signature.

Minhashing Example

1	4	3
3	2	4
7	1	7
6	3	6
2	6	1
5	7	2
4	5	5

Input matrix

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Surprising Property

- The probability (over all permutations of the rows) that $h(C_1) = h(C_2)$ is the same as $Sim(C_1, C_2)$.
- Both are $a/(a+b+c)$!
- Why?
  - Look down columns C_1 and C_2 until we see a 1.
  - If it's a type-*a* row, then $h(C_1) = h(C_2)$. If a type-*b* or type-*c* row, then not.

Similarity for Signatures

- The *similarity of signatures* is the fraction of the rows in which they agree.
  - Remember, each row corresponds to a permutation or "hash function."

Min Hashing – Example

1	4	3
3	2	4
7	1	7
6	3	6
2	6	1
5	7	2
4	5	5

Input matrix

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.67	1.00	0	0

Minhash Signatures

- Pick (say) 100 random permutations of the rows.
- Think of *Sig* (C) as a column vector.
- Let *Sig* (C)[i] = according to the *i* th permutation, the number of the first row that has a 1 in column *C*.

Implementation --- (1)

- Number of rows = 1 billion (say).
- Hard to pick a random permutation from 1…billion.
- Representing a random permutation requires 1 billion entries.
- Accessing rows in permuted order is tough!
  - The number of passes would be prohibitive.

Implementation --- (2)

1. Pick (say) 100 hash functions.
2. For each column *c* and each hash function h_i, keep a "slot" $M(i, c)$ for that minhash value.

Implementation --- (3)

for each row r

  for each column c

   if c has 1 in row r

    for each hash function h_i **do**

     if $h_i(r)$ is a smaller value than $M(i, c)$ **then**

      $M(i, c) := h_i(r)$

- Needs only one pass through the data.

Example

Row	C1	C2
1	1	0
2	0	1
3	1	1
4	1	0
5	0	1

$h(x) = x \bmod 5$
$g(x) = 2x+1 \bmod 5$

	Sig1	Sig2
$h(1) = 1$	1	-
$g(1) = 3$	3	-
$h(2) = 2$	1	2
$g(2) = 0$	3	0
$h(3) = 3$	1	2
$g(3) = 2$	2	0
$h(4) = 4$	1	2
$g(4) = 4$	2	0
$h(5) = 0$	1	0
$g(5) = 1$	2	0