Challenge Problem Set 3

May 12, 2010

1 Problem 1. (5 points)

Let S be the start symbol. Variables A, B, and C generate strings of L that end in 0, 1, and 2, respectively. The productions are:
S -> A | B | C | ϵ
A -> B0 | C0 | 0
B -> A1 | C1 | 1
C -> A2 | B2 | 2

2 Problem 2. (5 points)

Consider the NFA $A = (Q, \Sigma, \delta, q_0, F)$, which corresponds to the language $L(A) = L$. We can construct a CFG $G = (Q, \Sigma, P, Q_{q_0})$ such that $L(G) = L$. For every NFA state q, there is a variable Q_q. If $\delta(q, a)$ contains state p, then there is a production $Q_q \rightarrow aQ_p$.

In this proof, we need to show by induction that if A accepts ω then G generates ω and vice versa.

If A accepts ω then G generates ω

Basis. If $|\omega| = 0$, then $\omega = \epsilon$. If A accepts ϵ from q_k, then $q_k \in F$. Then, it follows that we have the production $Q_{q_k} \rightarrow \epsilon$, and thus $Q_{q_k} \Rightarrow^* \epsilon$.

Induction. Suppose $|\omega| \geq 1$, and that the inductive hypothesis holds for strings of length $< |\omega|$. Then we can write $\omega = ax$. Since $\omega \in L(A)$, $\hat{\delta}(q_0, ax) \in F$. This then implies that $\hat{\delta}(\delta(q_0, a), x) \in F$. From the induction, we have $\delta(q_0, a) = q_i$ and $\hat{\delta}(q_i, x) \in F$. There is a corresponding variable $Q_{q_0} \rightarrow aQ_{q_i}$ and a variable $Q_{q_i} \Rightarrow^* x$. Hence, $Q_{q_0} \Rightarrow^* ax = \omega$, which indicates G generates ω.

If G generates string ω then A accepts ω

Basis. $|\omega| = 0$, then $\omega = \epsilon$. Since if $Q_{q_k} \rightarrow \epsilon$. From our construction above, $q_k \in F$. Then $\omega = \epsilon$ is accepted by A.

Induction. Suppose $|\omega| \geq 1$, and that the inductive hypothesis holds for strings of length $< |\omega|$. Then, we can write $\omega = ax$. We have $Q_{q_k} \Rightarrow aQ_{q_i} \Rightarrow^* ax$, where Q_{q_i} corresponds to $q_i \in \delta(q_0, a)$ such that $\hat{\delta}(q_i, x) \in F$. Since the inductive hypothesis holds for x, we have $\hat{\delta}(q_i, x) \in F$. $\hat{\delta}(q_i, x) = \hat{\delta}(\delta(q_0, a), x) = \hat{\delta}(q_0, w) \in F$. Hence, A accepts ω starting from q_0.

3 Problem 3. (5 points)

Part a) Consider the string $a + a + a$, we have following two left-most derivations for the same:

- *Derivation 1*: $E \Rightarrow E + E \Rightarrow a + E \Rightarrow a + E + E \Rightarrow a + a + E \Rightarrow a + a + a$
- *Derivation 2*: $E \Rightarrow E + E \Rightarrow E + E + E \Rightarrow a + E + E \Rightarrow a + a + E \Rightarrow a + a + a$

Hence this grammar is ambiguous.

Part b) The following is an equivalent un-ambiguous grammar.

$$E \rightarrow E + T | T$$
$$T \rightarrow (E) | a$$

Points were deducted for:

- Not showing(or providing some reasoning) why a particular string is ambiguous.
- The modified grammar is ambiguous.
- The modified grammar though not ambiguous is not equivalent to the original grammar.