More About Turing Machines

"Programming Tricks"

Restrictions

Extensions

Closure Properties

Overview

- At first, the TM doesn't look very powerful.
  - Can it really do anything a computer can?
- We'll discuss "programming tricks" to convince you that it can simulate a real computer.

Overview – (2)

- We need to study restrictions on the basic TM model (e.g., tapes infinite in only one direction).
- Assuming a restricted form makes it easier to talk about simulating arbitrary TM's.
  - That's essential to exhibit a language that is not recursively enumerable.

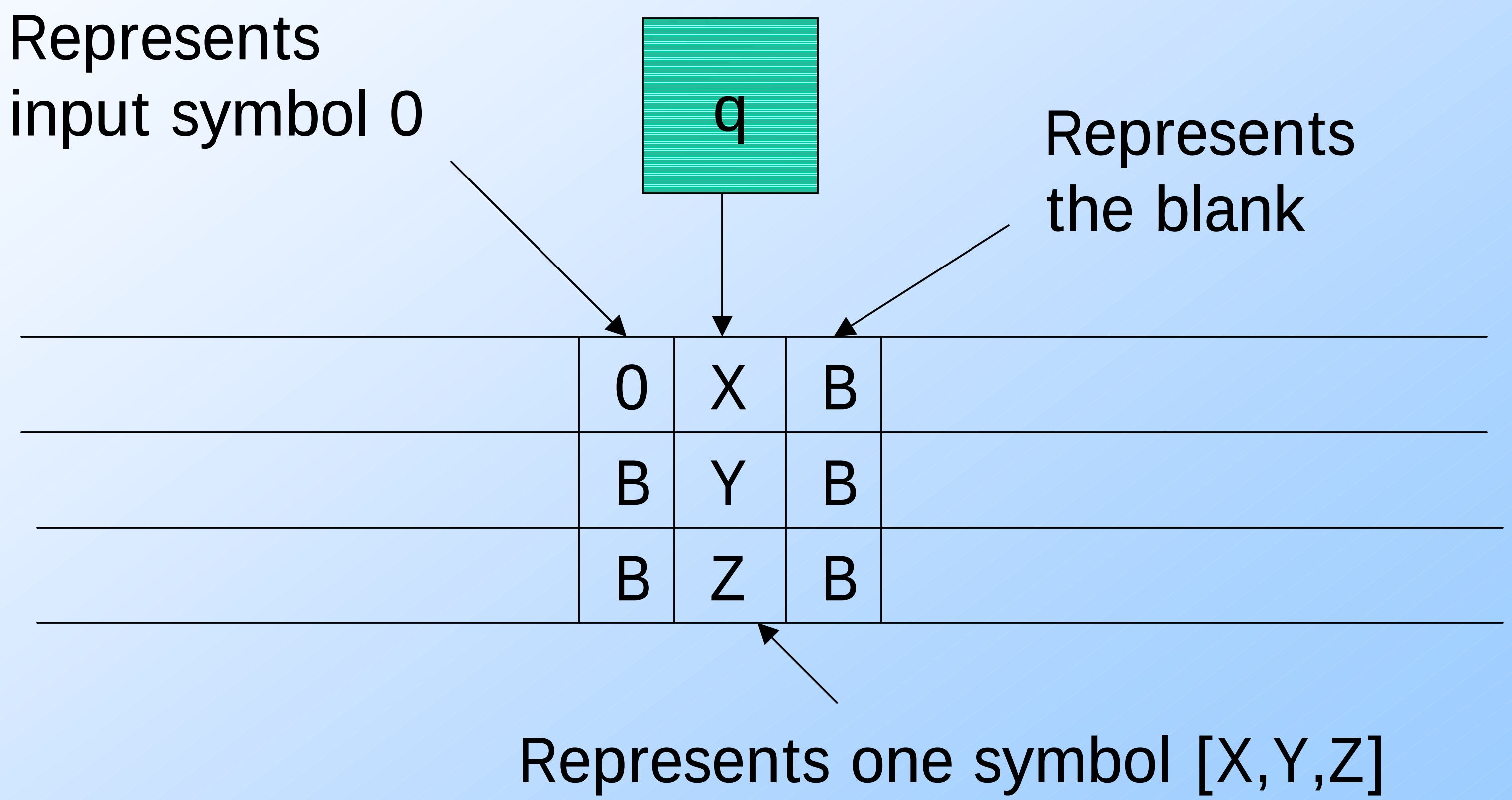
Overview – (3)

- We also need to study generalizations of the basic model.
- Needed to argue there is no more powerful model of what it means to "compute."
- Example: A nondeterministic TM with 50 six-dimensional tapes is no more powerful than the basic model.

Programming Trick: Multiple Tracks

- Think of tape symbols as vectors with k components.
- Each component chosen from a finite alphabet.
- Makes the tape appear to have k tracks.
- Let input symbols be blank in all but one track.

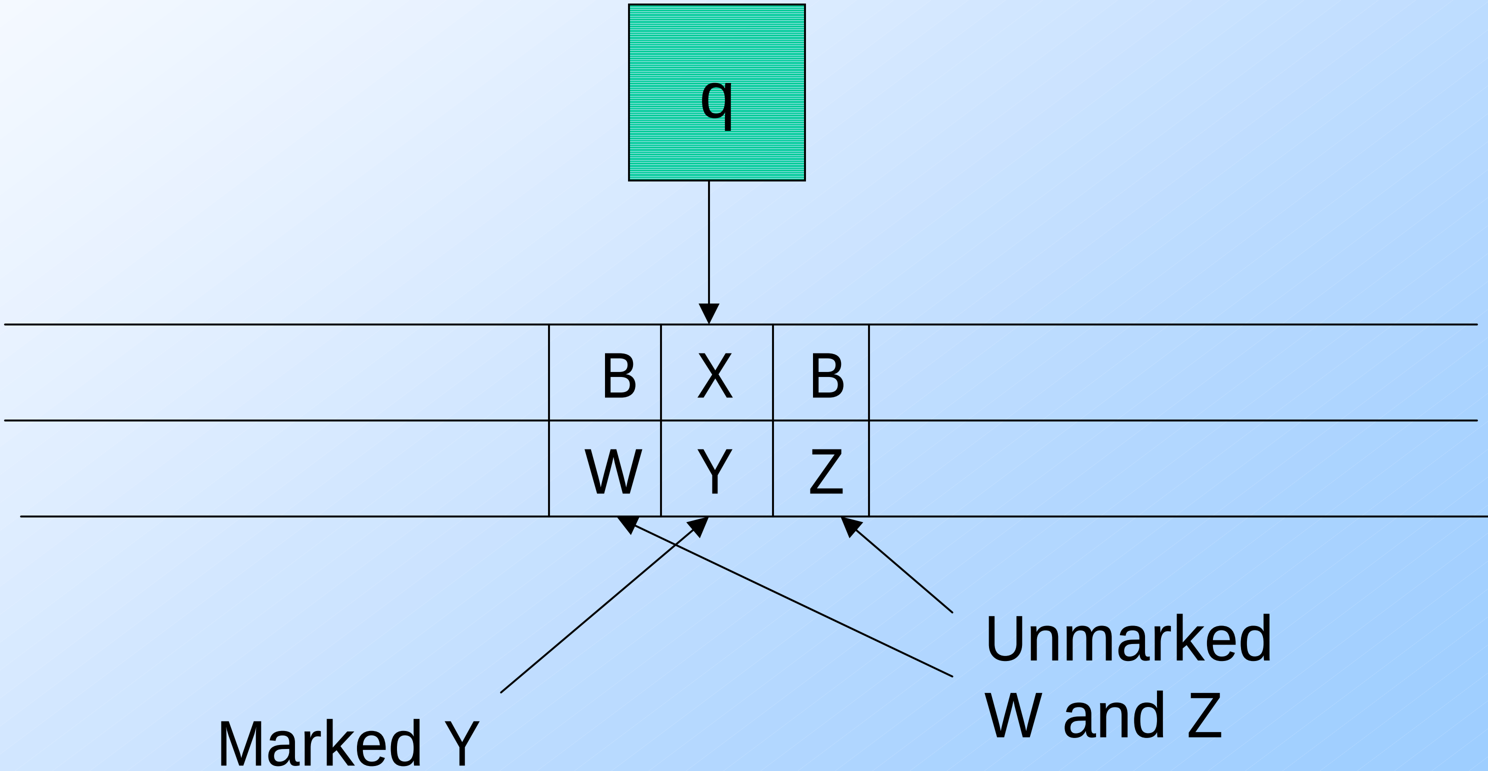
Picture of Multiple Tracks



Programming Trick: Marking

- A common use for an extra track is to *mark* certain positions.
- Almost all cells hold B (blank) in this track, but several hold special symbols (marks) that allow the TM to find particular places on the tape.

Marking



Programming Trick: Caching in the State

- The state can also be a vector.
- First component is the “control state.”
- Other components hold data from a finite alphabet.

Example: Using These Tricks

- This TM doesn't do anything terribly useful; it copies its input w infinitely.
- Control states:
  - q: Mark your position and remember the input symbol seen.
  - p: Run right, remembering the symbol and looking for a blank. Deposit symbol.
  - r: Run left, looking for the mark.

Example – (2)

- States have the form [x, Y], where x is q, p, or r and Y is 0, 1, or B.
  - Only p uses 0 and 1.
- Tape symbols have the form [U, V].
  - U is either X (the "mark") or B.
  - V is 0, 1 (the input symbols) or B.
  - [B, B] is the TM blank; [B, 0] and [B, 1] are the inputs.

The Transition Function

- Convention: *a* and *b* each stand for "either 0 or 1."
- δ([q,B], [B,a]) = ([p,a], [X,a], R).
  - In state q, copy the input symbol under the head (i.e., *a*) into the state.
  - Mark the position read.
  - Go to state p and move right.

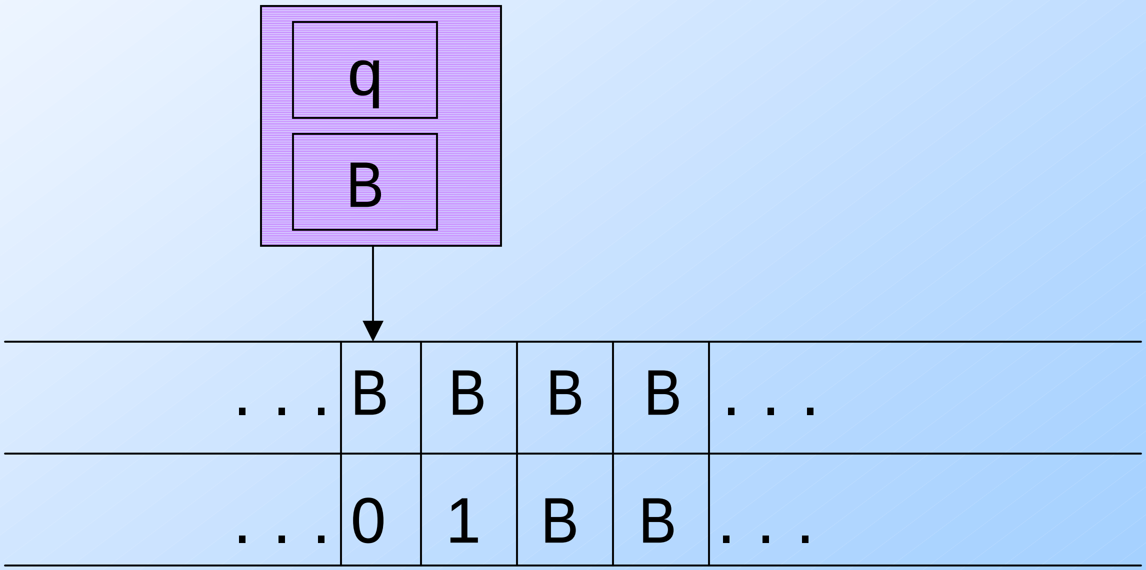
Transition Function – (2)

- δ([p,a], [B,b]) = ([p,a], [B,b], R).
  - In state p, search right, looking for a blank symbol (not just B in the mark track).
- δ([p,a], [B,B]) = ([r,B], [B,a], L).
  - When you find a B, replace it by the symbol (*a*) carried in the "cache."
  - Go to state r and move left.

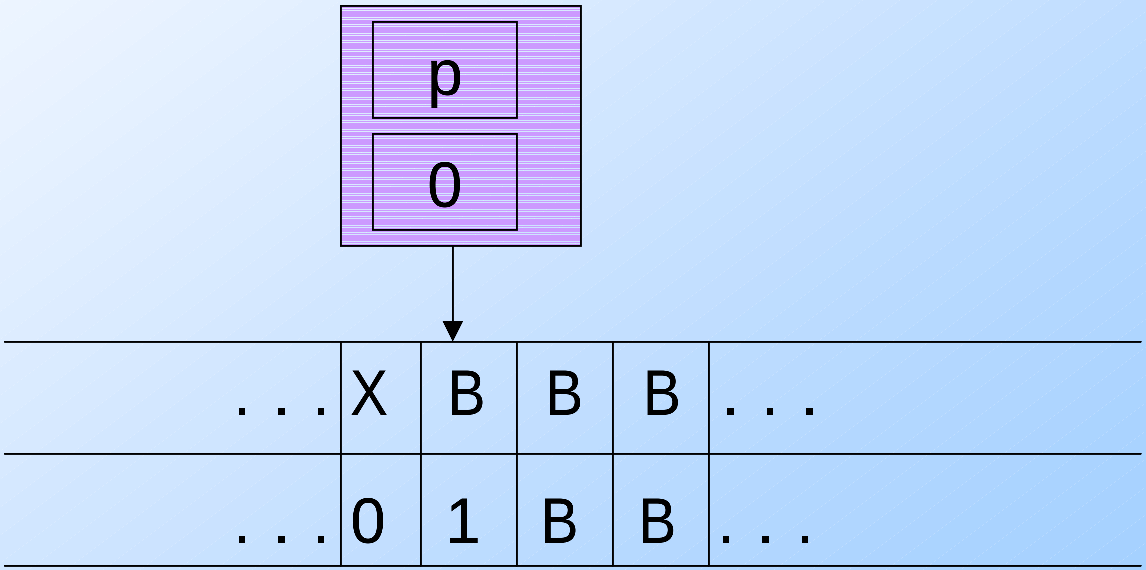
Transition Function – (3)

- $\delta([r,B], [B,a]) = ([r,B], [B,a], L)$.
  - In state r, move left, looking for the mark.
- $\delta([r,B], [X,a]) = ([q,B], [B,a], R)$.
  - When the mark is found, go to state q and move right.
  - But remove the mark from where it was.
  - q will place a new mark and the cycle repeats.

Simulation of the TM



Simulation of the TM



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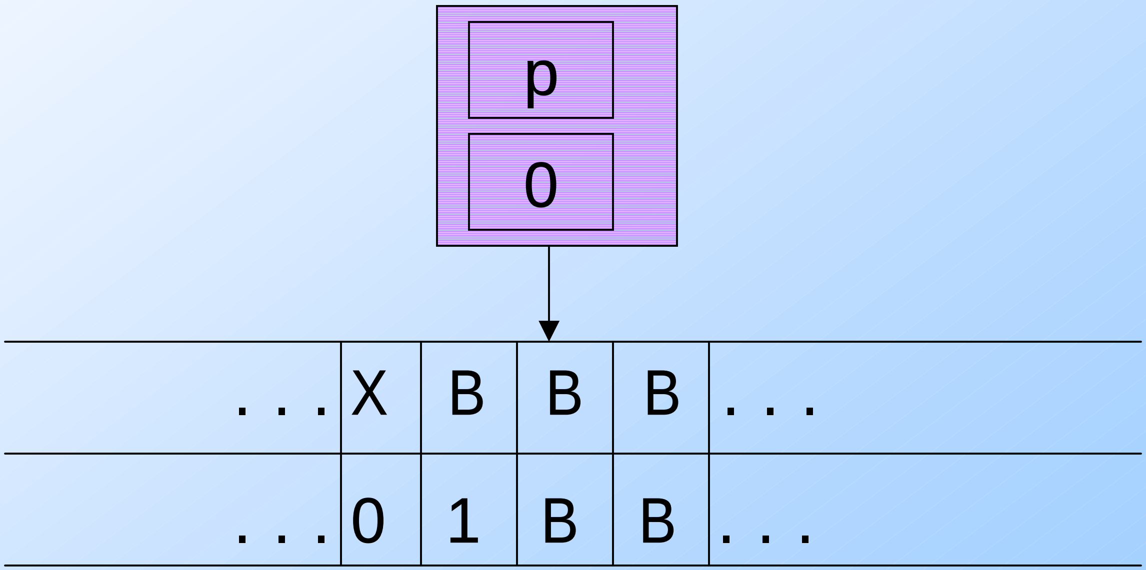

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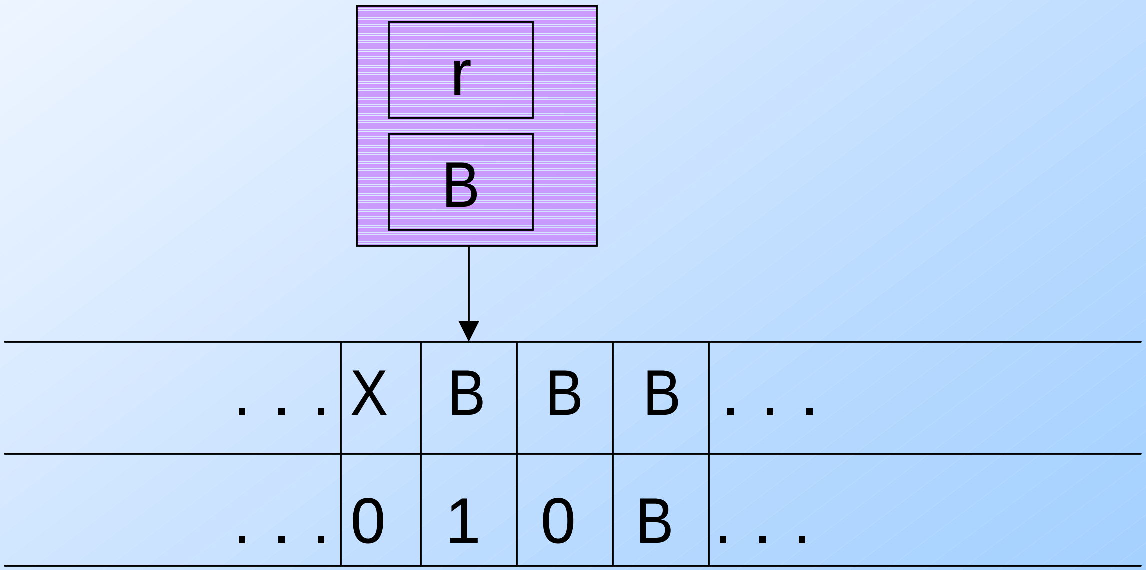

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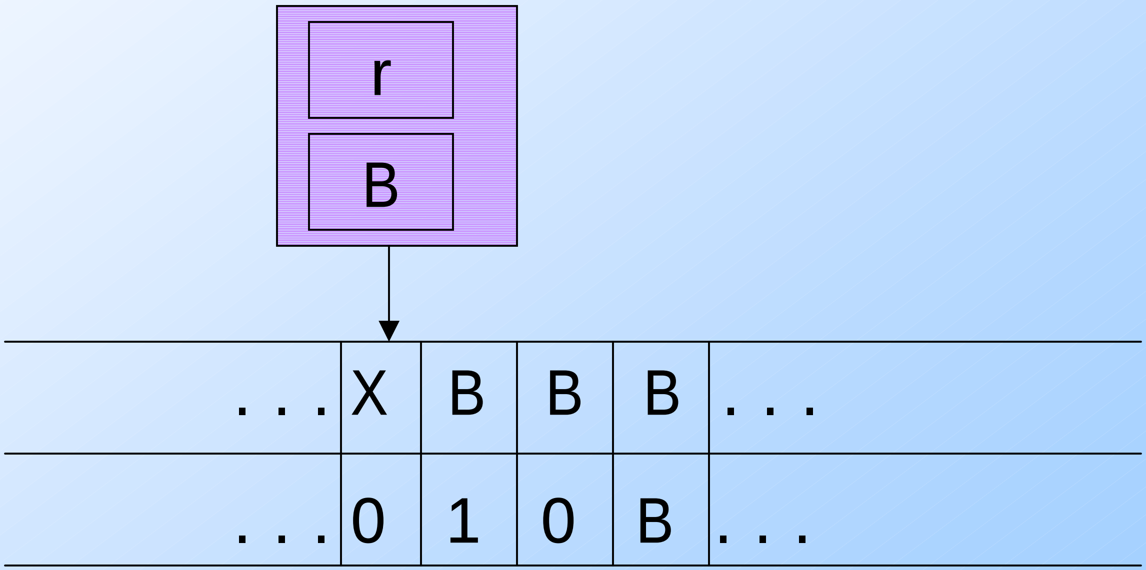

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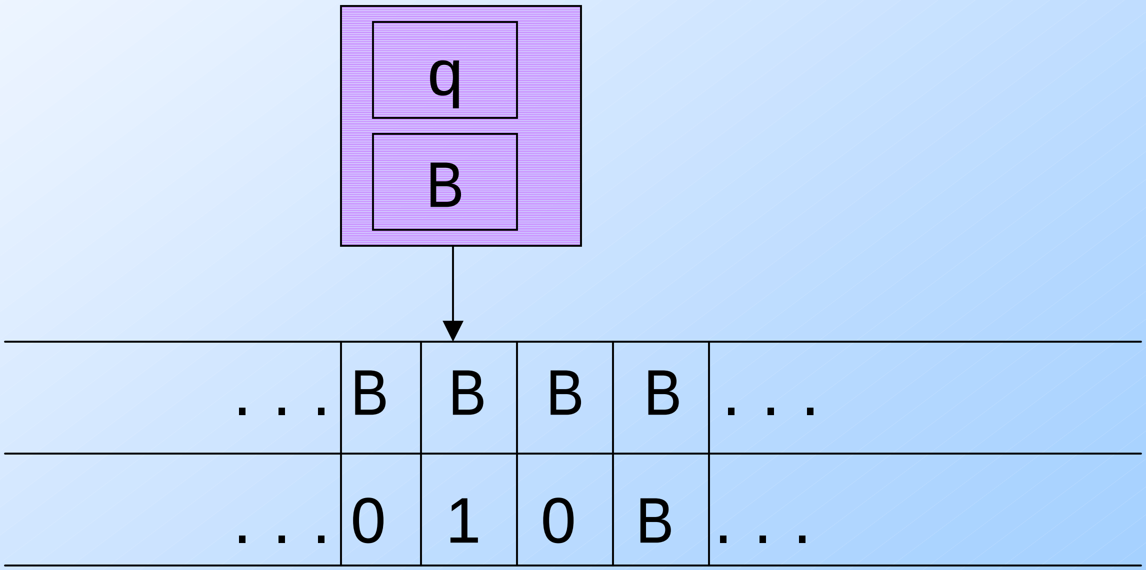
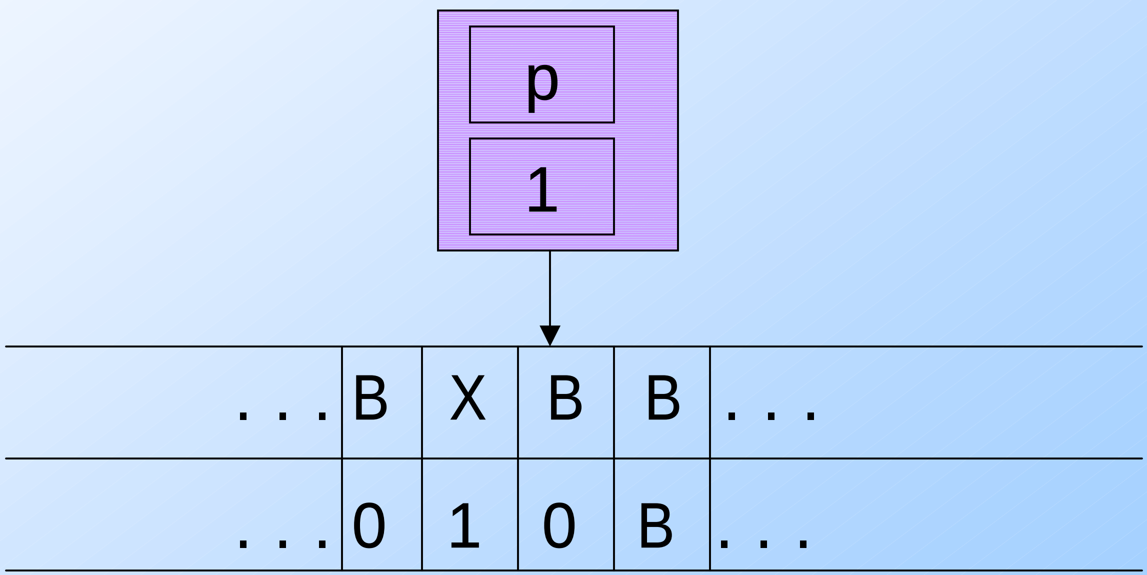


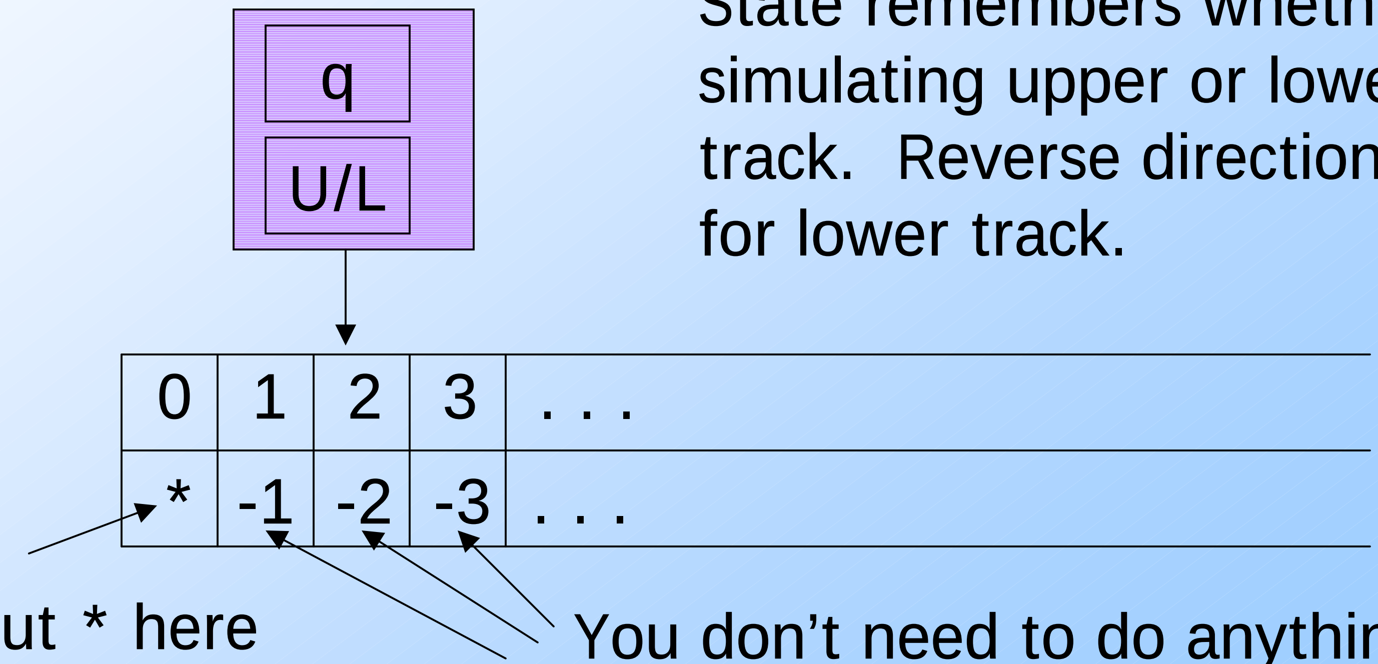
Simulation of the TM



Semi-infinite Tape

- We can assume the TM never moves left from the initial position of the head.
- Let this position be 0; positions to the right are 1, 2, ... and positions to the left are –1, –2, ...
- New TM has two tracks.
  - Top holds positions 0, 1, 2, ...
  - Bottom holds a marker, positions –1, –2, ...

Simulating Infinite Tape by Semi-infinite Tape



State remembers whether simulating upper or lower track. Reverse directions for lower track.

Put * here at the first move

You don’t need to do anything, because these are initially B.

More Restrictions – Read in Text

- Two stacks can simulate one tape.
  - One holds positions to the left of the head; the other holds positions to the right.
- In fact, by a clever construction, the two stacks to be *counters* = only two stack symbols, one of which can only appear at the bottom.

Factoid: Invented by Pat Fischer, whose main claim to fame is that he was a victim of the Unabomber.

Extensions

- More general than the standard TM.
- But still only able to define the RE languages.
  1. Multitape TM.
  2. Nondeterministic TM.
  3. Store for key-value pairs.

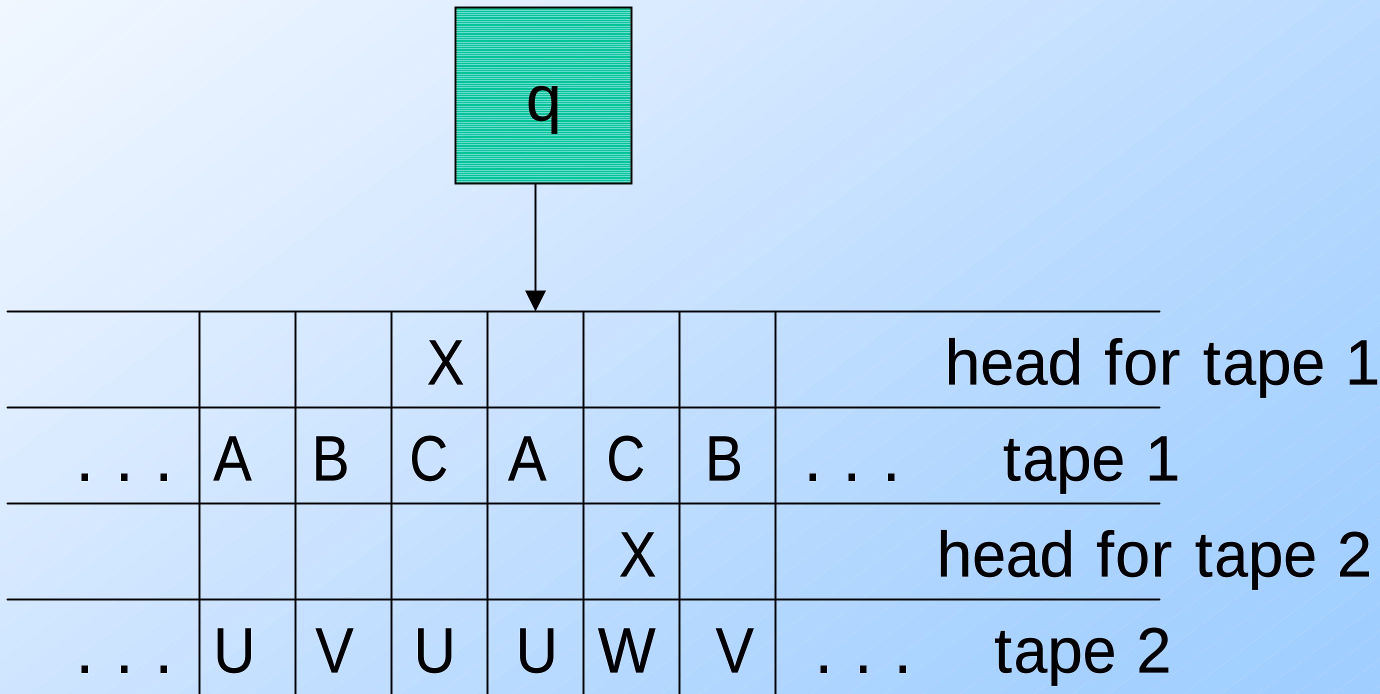
Multitape Turing Machines

- Allow a TM to have k tapes for any fixed k.
- Move of the TM depends on the state and the symbols under the head for each tape.
- In one move, the TM can change state, write symbols under each head, and move each head independently.

Simulating k Tapes by One

- Use 2k tracks.
- Each tape of the k-tape machine is represented by a track.
- The head position for each track is represented by a mark on an additional track.

Picture of Multitape Simulation



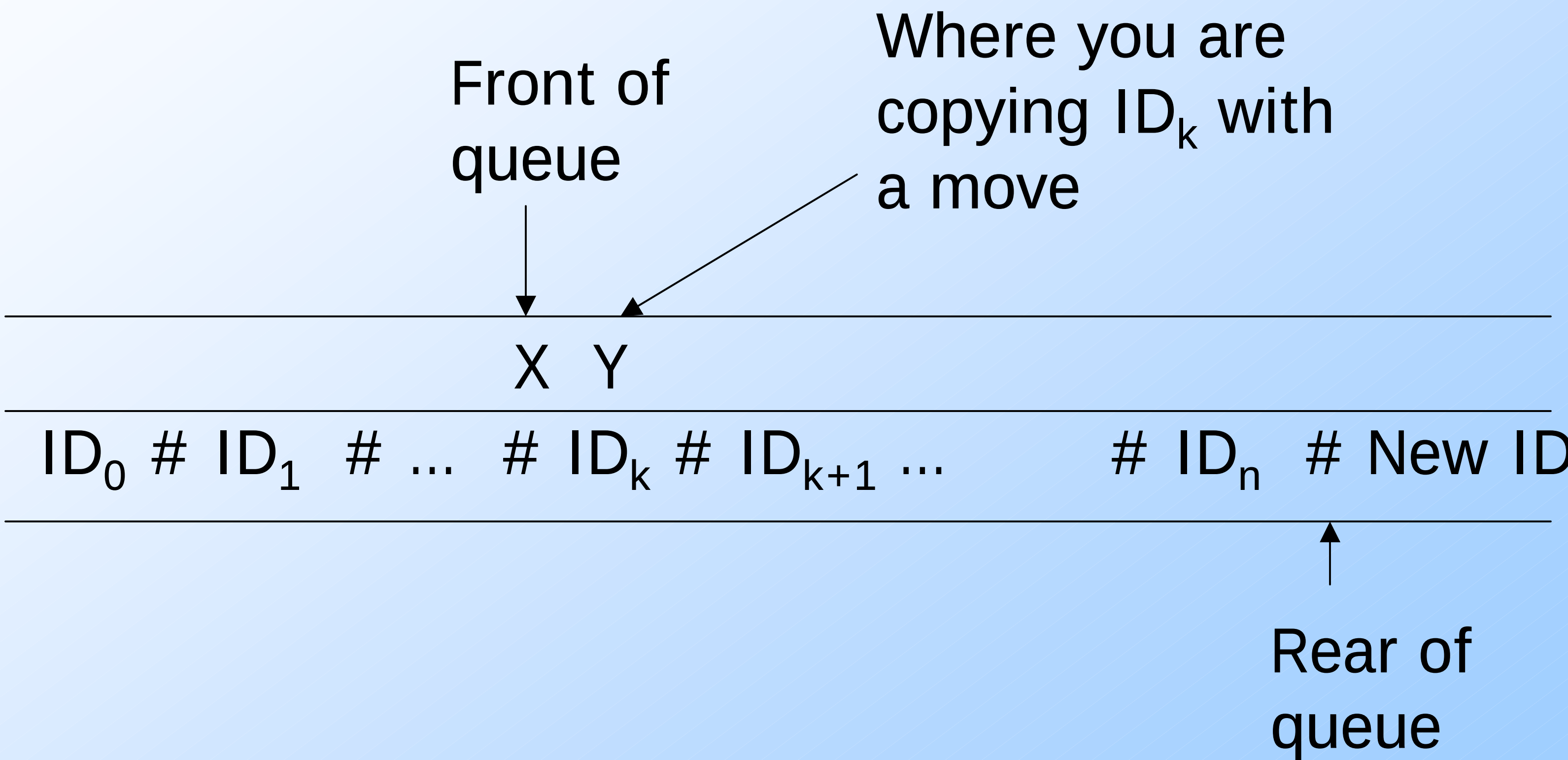
Nondeterministic TM's

- Allow the TM to have a choice of move at each step.
  - Each choice is a state-symbol-direction triple, as for the deterministic TM.
- The TM accepts its input if any sequence of choices leads to an accepting state.

Simulating a NTM by a DTM

- The DTM maintains on its tape a queue of ID's of the NTM.
- A second track is used to mark certain positions:
  1. A mark for the ID at the head of the queue.
  2. A mark to help copy the ID at the head and make a one-move change.

Picture of the DTM Tape



Operation of the Simulating DTM

- The DTM finds the ID at the current front of the queue.
- It looks for the state in that ID so it can determine the moves permitted from that ID.
- If there are m possible moves, it creates m new ID's, one for each move, at the rear of the queue.

Operation of the DTM – (2)

- The m new ID's are created one at a time.
- After all are created, the marker for the front of the queue is moved one ID toward the rear of the queue.
- However, if a created ID has an accepting state, the DTM instead accepts and halts.

Why the NTM -> DTM Construction Works

- There is an upper bound, say k, on the number of choices of move of the NTM for any state/symbol combination.
- Thus, any ID reachable from the initial ID by n moves of the NTM will be constructed by the DTM after constructing at most $(k^{n+1}-k)/(k-1)$ ID's.

Sum of $k+k^2+\ldots+k^n$

Why? – (2)

- If the NTM accepts, it does so in some sequence of n choices of move.
- Thus the ID with an accepting state will be constructed by the DTM in some large number of its own moves.
- If the NTM does not accept, there is no way for the DTM to accept.

Taking Advantage of Extensions

- We now have a really good situation.
- When we discuss construction of particular TM's that take other TM's as input, we can assume the input TM is as simple as possible.
  - E.g., one, semi-infinite tape, deterministic.
- But the simulating TM can have many tapes, be nondeterministic, etc.

Real Computers

- Recall that, since a real computer has finite memory, it is in a sense weaker than a TM.
- Imagine a computer with an infinite store for name-value pairs.
  - Generalizes an address space.

Simulating a Name-Value Store by a TM

- The TM uses one of several tapes to hold an arbitrarily large sequence of name-value pairs in the format #name*value#...
- Mark, using a second track, the left end of the sequence.
- A second tape can hold a name whose value we want to look up.

Lookup

- Starting at the left end of the store, compare the lookup name with each name in the store.
- When we find a match, take what follows between the * and the next # as the value.

Insertion

- Suppose we want to insert name-value pair (n, v), or replace the current value associated with name n by v.
- Perform lookup for name n.
- If not found, add n*v# at the end of the store.

Insertion – (2)

- If we find #n*v'#, we need to replace v' by v.
- If v is shorter than v', you can leave blanks to fill out the replacement.
- But if v is longer than v', you need to make room.

Insertion – (3)

- Use a third tape to copy everything from the first tape at or to the right of v'.
- Mark the position of the * to the left of v' before you do.
- Copy from the third tape to the first, leaving enough room for v.
- Write v where v' was.

Closure Properties of Recursive and RE Languages

- Both closed under union, concatenation, star, reversal, intersection, inverse homomorphism.
- Recursive closed under difference, complementation.
- RE closed under homomorphism.

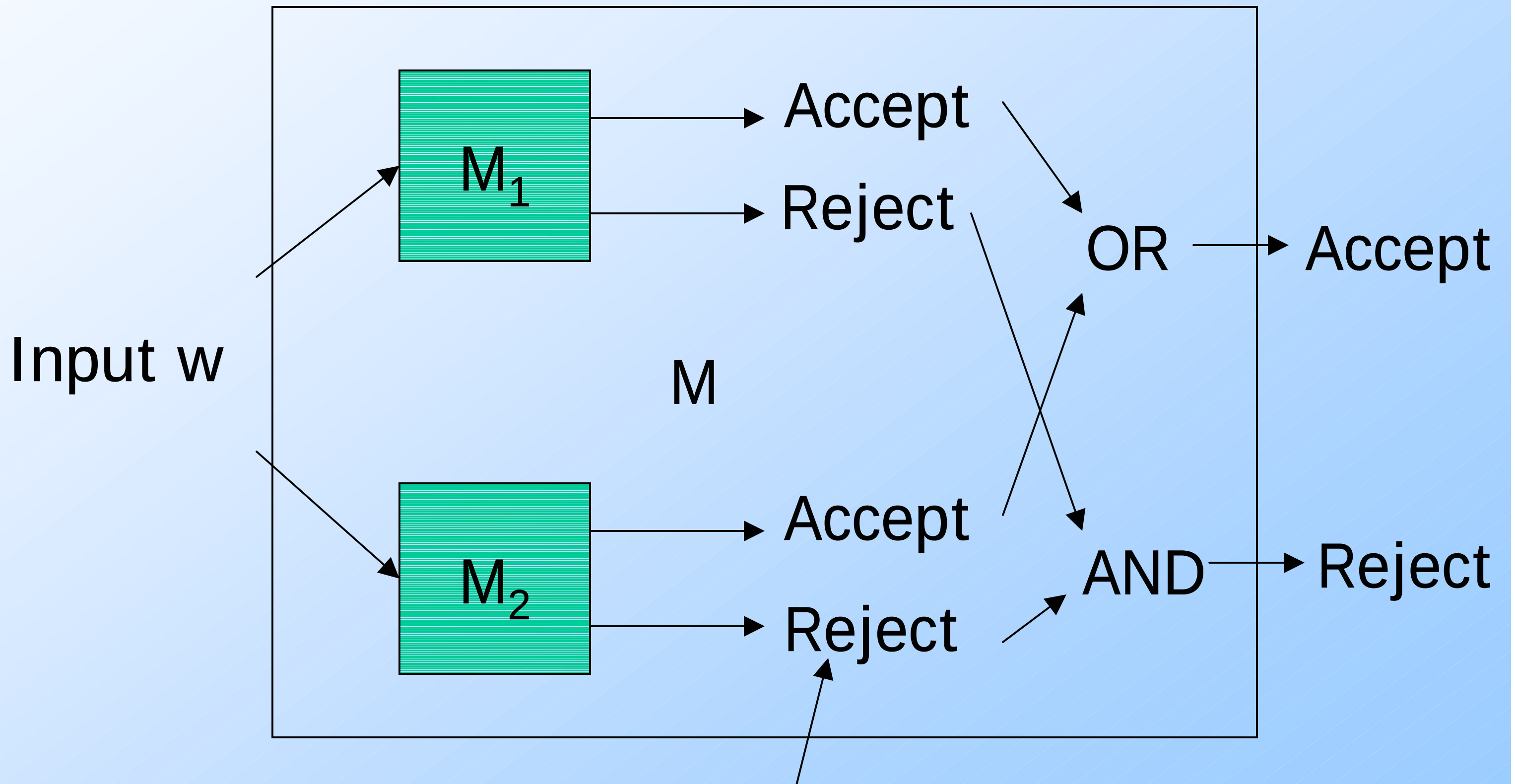
Union

- Let $L_1 = L(M_1)$ and $L_2 = L(M_2)$.
- Assume M_1 and M_2 are single-semi-infinite-tape TM's.
- Construct 2-tape TM M to copy its input onto the second tape and simulate the two TM's M_1 and M_2 each on one of the two tapes, "in parallel."

Union – (2)

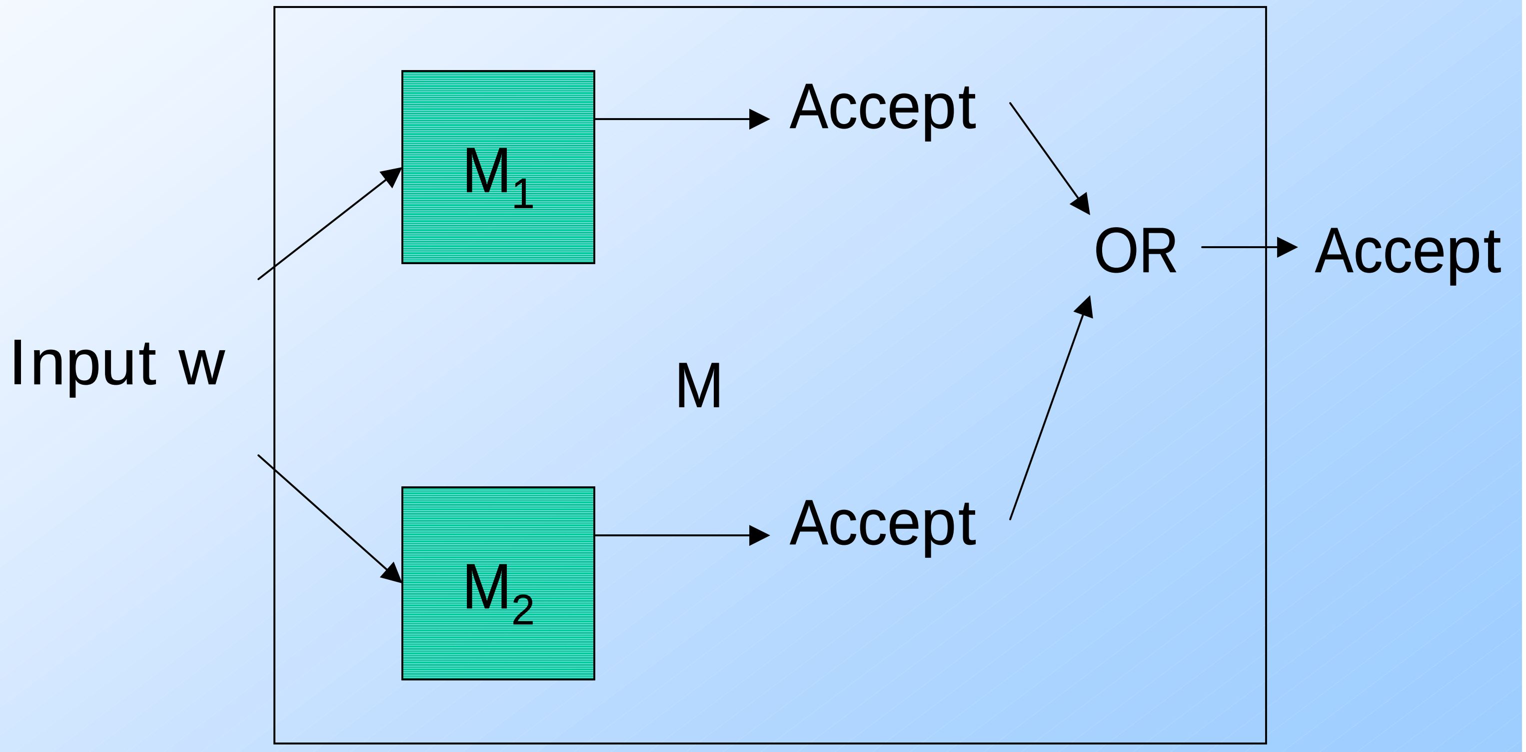
- Recursive languages: If M_1 and M_2 are both algorithms, then M will always halt in both simulations.
- Accept if either accepts.
- RE languages: accept if either accepts, but you may find both TM's run forever without halting or accepting.

Picture of Union/Recursive

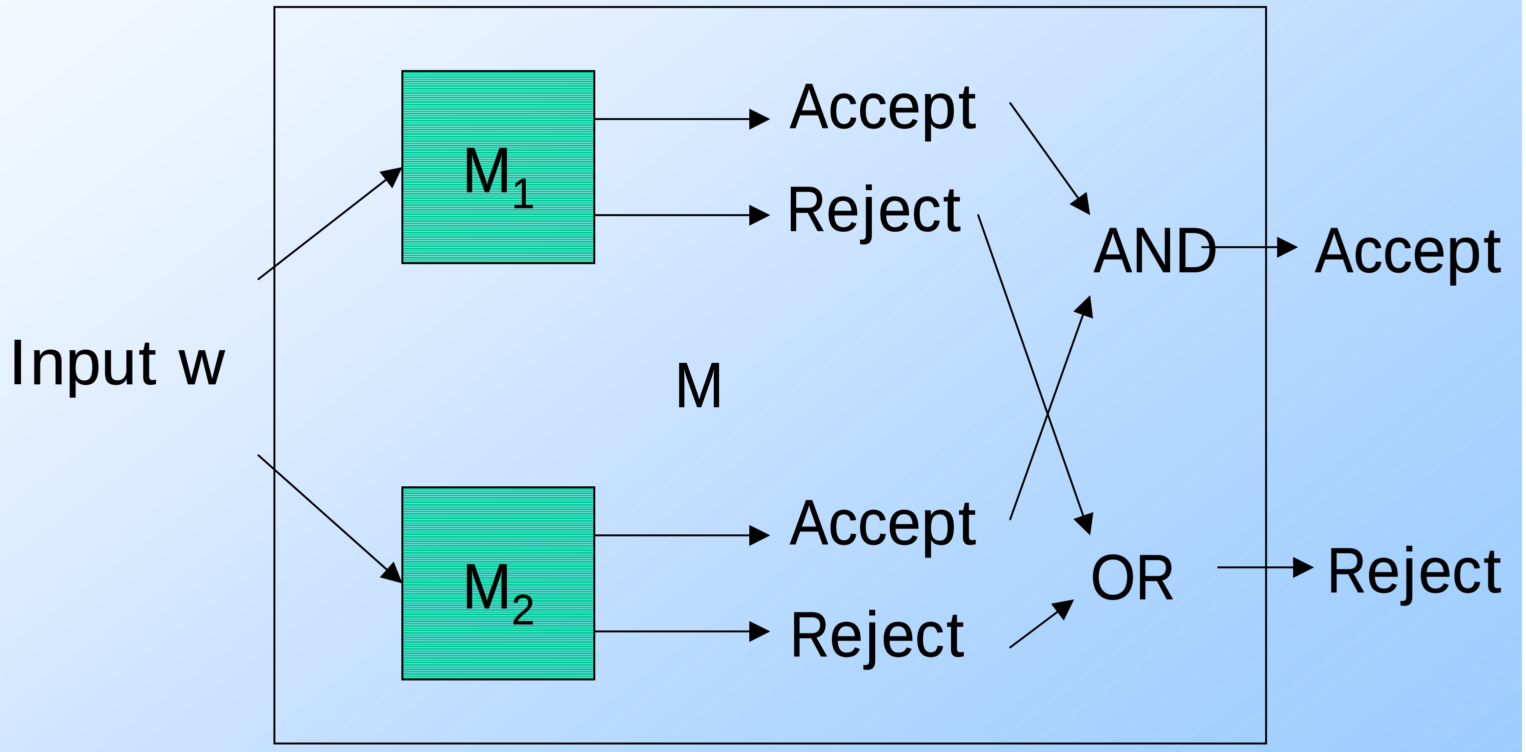


Remember: = "halt without accepting

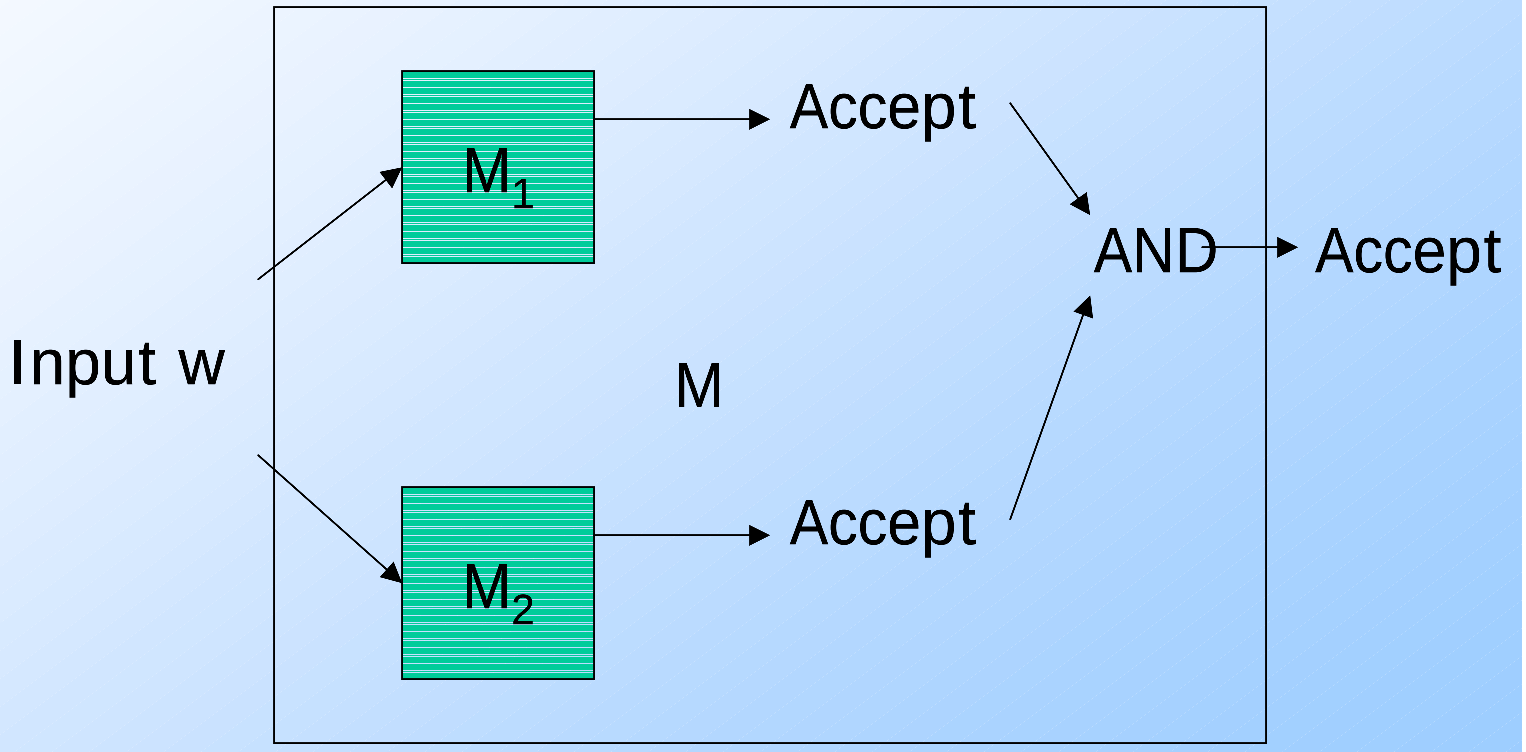
Picture of Union/RE



Intersection/Recursive – Same Idea



Intersection/RE



Difference, Complement

- Recursive languages: both TM's will eventually halt.
- Accept if M_1 accepts and M_2 does not.
  - Corollary: Recursive languages are closed under complementation.
- RE Languages: can't do it; M_2 may never halt, so you can't be sure input is in the difference.

Concatenation/RE

- Let $L_1 = L(M_1)$ and $L_2 = L(M_2)$.
- Assume M_1 and M_2 are single-semi-infinite-tape TM's.
- Construct 2-tape Nondeterministic TM M:
  1. Guess a break in input w = xy.
  2. Move y to second tape.
  3. Simulate M_1 on x, M_2 on y.
  4. Accept if both accept.

Concatenation/Recursive

- Can't use a NTM.
- Systematically try each break w = xy.
- M_1 and M_2 will eventually halt for each break.
- Accept if both accept for any one break.
- Reject if all breaks tried and none lead to acceptance.

Star

- Same ideas work for each case.
- RE: guess many breaks, accept if M_1 accepts each piece.
- Recursive: systematically try all ways to break input into some number of pieces.

Reversal

- Start by reversing the input.
- Then simulate TM for L to accept w if and only w^R is in L.
- Works for either Recursive or RE languages.

Inverse Homomorphism

- Apply h to input w.
- Simulate TM for L on h(w).
- Accept w iff h(w) is in L.
- Works for Recursive or RE.

Homomorphism/RE

- Let $L = L(M_1)$.
- Design NTM M to take input w and guess an x such that $h(x) = w$.
- M accepts whenever M_1 accepts x.
- Note: won't work for Recursive languages.