# CS347

Lecture 8

May 7, 2001

# Today’s topic

- Clustering documents

# Why cluster documents

- Given a corpus, partition it into groups of related docs
  – Recursively, can induce a tree of topics
- Given the set of docs from the results of a search (say ***jaguar***), partition into groups of related docs
  – semantic disambiguation

# Results list clustering example

- Cluster 1:
  - Jaguar Motor Cars' home page
  - Mike's XJS resource page
  - Vermont Jaguar owners' club

- Cluster 2:
  - Big cats
  - My summer safari trip
  - Pictures of jaguars, leopards and lions

- Cluster 3:
  - Jacksonville Jaguars' Home Page
  - AFC East Football Teams

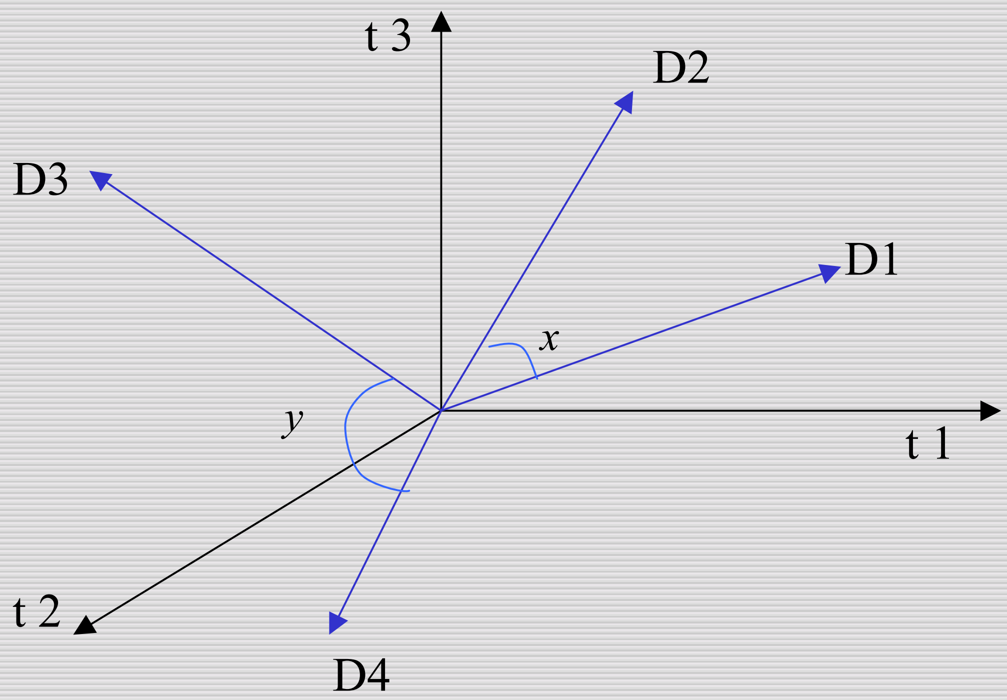
# What makes docs "related"?

- Ideal: semantic similarity.
- Practical: statistical similarity
  - We will use cosine similarity.
  - Docs as vectors.
  - For many algorithms, easier to think in terms of a *distance* (rather than <u>similarity</u>) between docs.
  - We will describe algorithms in terms of cosine distance

# Recall doc as vector

- Each doc *j* is a vector of *tf*×*idf* values, one component for each term.
- Can normalize to unit length.
- So we have a vector space
  - terms are axes
  - *n* docs live in this space
  - even with stemming, may have 10000+ dimensions

# Intuition




Postulate: Documents that are “close together” in vector space talk about the same things.

# Cosine similarity

Cosine similarity of $D_j, D_k$:

$$sim(D_j, D_k) = \sum_{i=1}^{m} w_{ij} \times w_{ik}$$

Aka normalized inner product.

# Two flavors of clustering

- Given *n* docs and a positive integer *k*, partition docs into *k* (disjoint) subsets.
- Given docs, partition into an "appropriate" number of subsets.
  - E.g., for query results - ideal value of *k* not known up front.
- Can usually take an algorithm for one flavor and convert to the other.

# Cluster centroid

- Centroid of a cluster = average of vectors in a cluster - is a vector.
  - Need not be a doc.
- Centroid of (1,2,3); (4,5,6); (7,2,6) is (4,3,5).

# Outliers in centroid computation

- Ignore outliers when computing centroid.
  - What is an outlier?
  - Distance to centroid > M × average.

Say 10.

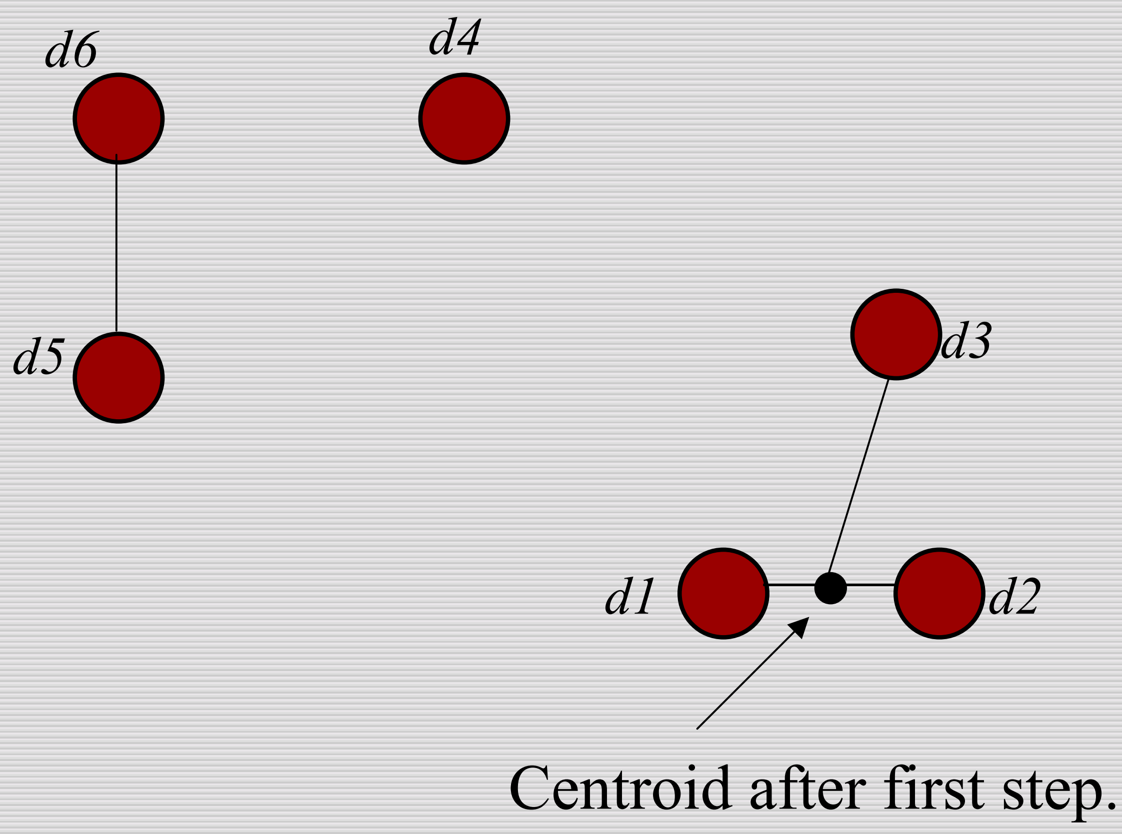
# Agglomerative clustering

- Given target number of clusters *k*.
- Initially, each doc viewed as a cluster
  - start with *n* clusters;
- Repeat:
  - **while** there are > *k* clusters, find the "closest pair" of clusters and merge them.

# "Closest pair" of clusters

- Many variants to defining closest pair of clusters.
- Closest pair $\Leftrightarrow$ two clusters whose centroids are the most cosine-similar.

# Example; $n=6$, $k=3$



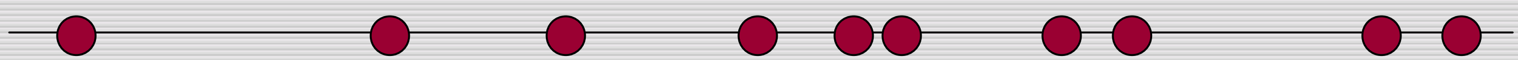
# Issues

- Have to discover closest pairs
  - compare all pairs?
    - $n^3$ cosine similarity computations.
    - Avoid: recall techniques from lecture 4.
  - points are changing as centroids change.
- Changes at each step are not localized
  - on a large corpus, memory management becomes an issue.

How would you adapt sampling/pre-grouping?

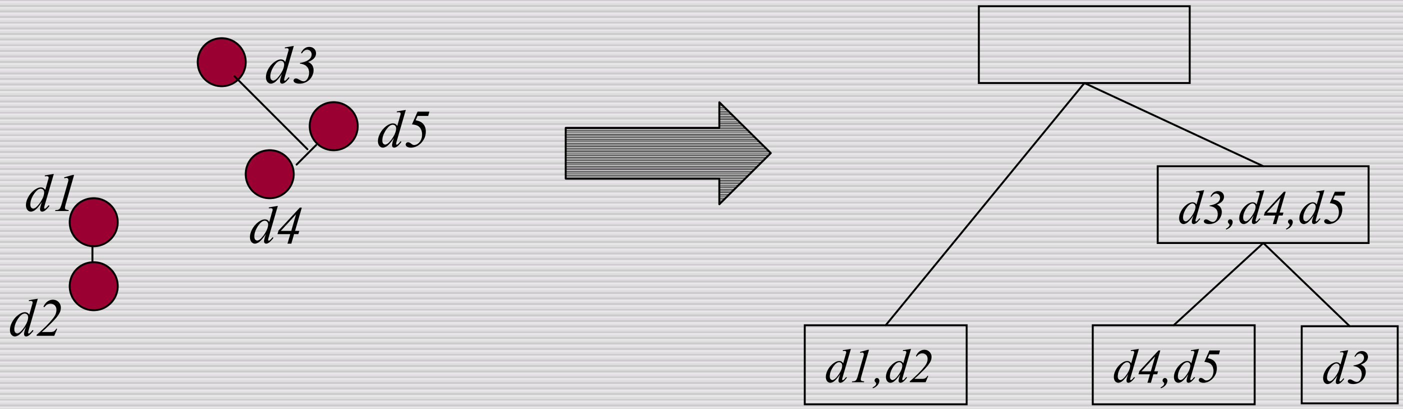
# Exercise

- Consider agglomerative clustering on $n$ points on a line. Explain how you could avoid $n^3$ distance computations - how many will your scheme use?



# Hierarchical clustering

- As clusters *agglomerate*, docs likely to fall into a hieararchy of “topics” or concepts.



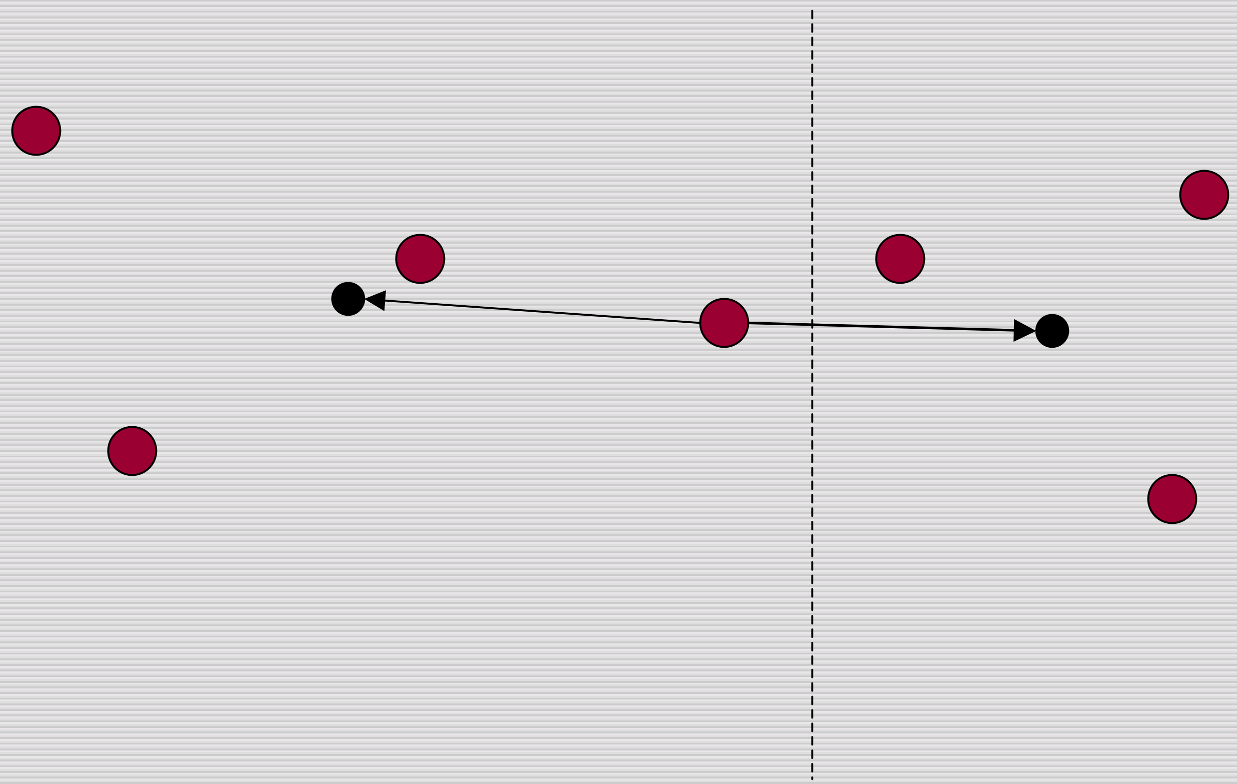
# Different algorithm: *k*-means

- Iterative algorithm.
- More locality within each iteration.
- Hard to get good bounds on the number of iterations.

# Basic iteration

- At the start of the iteration, we have *k* centroids.
  - Need not be docs, just some *k* points.
- Each doc assigned to the nearest centroid.
- All docs assigned to the same centroid are averaged to compute a new centroid;
  - thus have *k* new centroids.

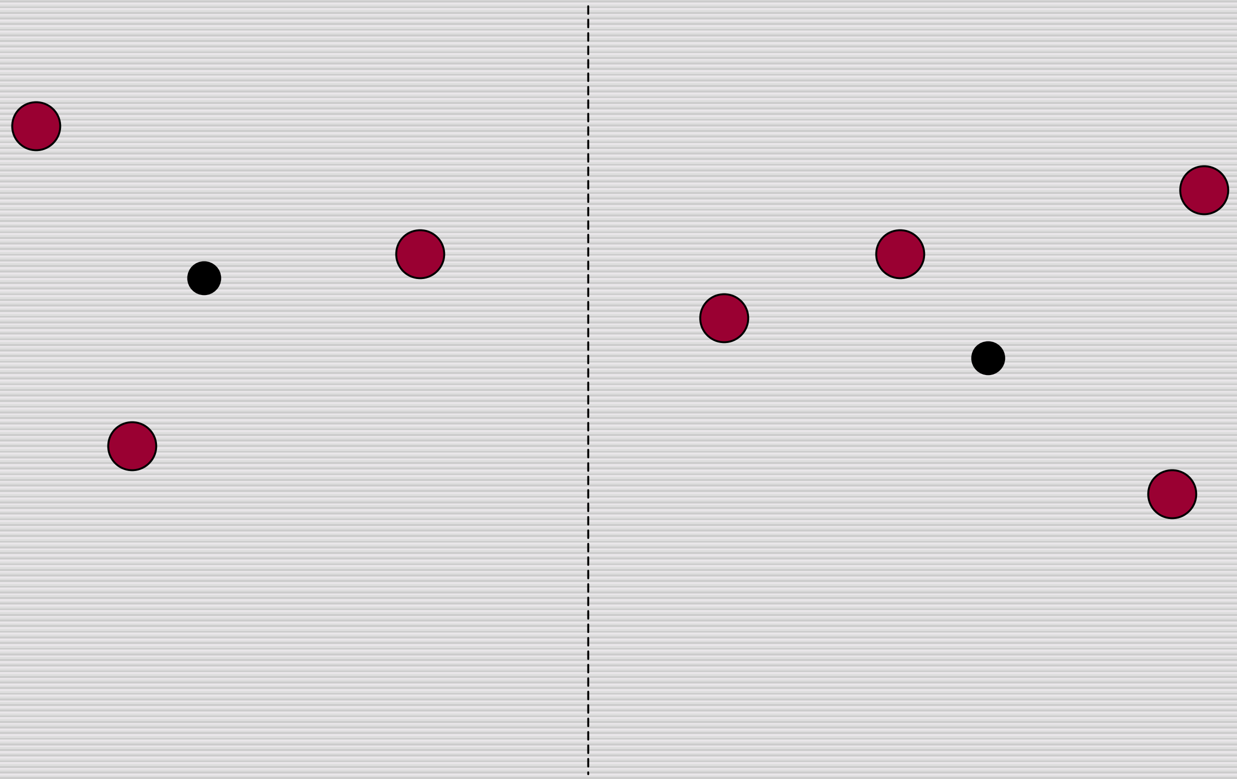
# Iteration example



Docs

Current centroids

# Iteration example



Docs
New centroids

# *k*-means clustering

- Begin with *k* docs as centroids
  - could be any *k* docs, but *k* random docs are better.
- Repeat Basic Iteration until termination condition satisfied.

Why?

# Termination conditions

- Several possibilities, e.g.,
  - A fixed number of iterations.
  - Centroid positions don’t change.

Does this mean that the docs in a cluster are unchanged?

# Convergence

- Why should the *k*-means algorithm ever reach a *fixed point*?
  - A state in which clusters don't change.
- *k*-means is a special case of a general procedure known as the *EM algorithm*.
  - Under reasonable conditions, known to converge.
  - Number of iterations could be large.

# Exercise

- Consider running 2-means clustering on a corpus, each doc of which is from one of two different languages. What are the two clusters we would expect to see?
- Is agglomerative clustering likely to produce different results?

# Multi-lingual docs

- Canadian/Belgian government docs.
- Every doc in English and equivalent French.
  - Cluster by concepts rather than language.
  - Cross-lingual retrieval.

# $k$ not specified in advance

- Say, the results of a query.
- Solve an optimization problem: penalize having lots of clusters
  - compressed summary of list of docs.
- Tradeoff between having more clusters (better focus within each cluster) and having too many clusters

# *k* not specified in advance

- Given a clustering, define the Benefit for a doc to be the cosine similarity to its centroid
- Define the Total Benefit to be the sum of the individual doc Benefits.

Why is there always a clustering of Total Benefit $n$?

# Penalize lots of clusters

- For each cluster, we have a Cost $C$.
- Thus for a clustering with $k$ clusters, the Total Cost is $kC$.
- Define the Value of a cluster to be =
  - Total Benefit - Total Cost.
- Find the clustering of highest Value, over all choices of $k$.

# Back to agglomerative clustering

- In a run of agglomerative clustering, we can try all values of $k=n,n-1,n-2, \ldots 1$.
- At each, we can measure our Value, then pick the best choice of $k$.

# Exercises

- Suppose a run of agglomerative clustering finds $k=7$ to have the highest Value amongst all $k$. Have we found the highest-Value clustering amongst all clusterings with $k=7$?

# Using clustering in applications

# Clustering to speed up scoring

- From Lecture 4, recall sampling and pre-grouping
  - Wanted to find, given a query *Q,* the nearest docs in the corpus
  - Wanted to avoid computing cosine similarity of *Q* to each of n docs in the corpus.

# Sampling and pre-grouping
(Lecture 4)

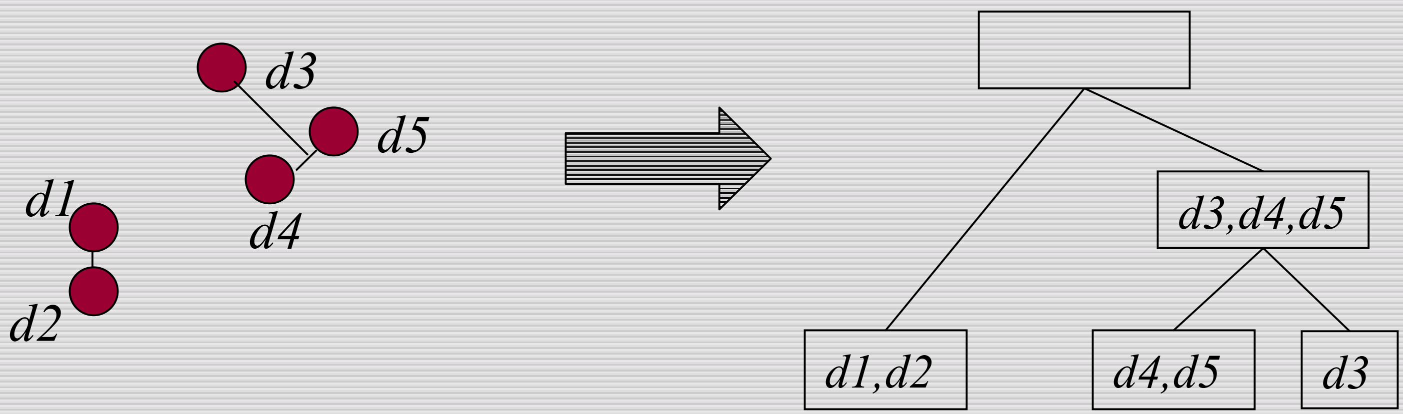
- First run a pre-processing phase:
  – pick $\sqrt{n}$ *docs* at random: call these *leaders*
  – For each other doc, pre-compute nearest leader
    - Docs attached to a leader: its *followers;*
    - Likely: each leader has ~ $\sqrt{n}$ followers.
- Process a query as follows:
  – Given query *Q*, find its nearest *leader L*.
  – Seek nearest docs from among *L*'s followers.

# Instead of random leaders, cluster

- First run a pre-processing phase:
  - Cluster docs into $\sqrt{n}$ clusters.
  - For each cluster, its centroid is the *leader*.
- Process a query as follows:
  - Given query $Q$, find its nearest leader $L$.
  - Seek nearest docs from among $L$'s followers.

# Navigation structure

- Given a corpus, agglomerate into a hierarchy
- Throw away lower layers so you don't have *n* leaf topics each having a single doc.



# Navigation structure

- Deciding how much to throw away needs human judgement.
- Can also induce hierarchy top-down - e.g., use *k*-means, then recur on the clusters.
- Topics induced by clustering need human ratification.
- Need to address issues like partitioning at the top level by language.

# Major issue - labelling

- After clustering algorithm finds clusters - how can they be useful to the end user?
- Need pithy label for each cluster
  - In search results, say “Football” or “Car” in the ***jaguar*** example.
  - In topic trees, need navigational cues.
    - Often done by hand, a posteriori.

# Labeling

- Common heuristics - list 5-10 most frequent terms in the centroid vector.
  - Drop stop-words; stem.
- Differential labeling by frequent terms
  - Within the cluster "Computers", child clusters all have the word ***computer*** as frequent terms.
  - Discriminant analysis of centroids for peer clusters.

# Supervised vs. unsupervised learning

- Unsupervised learning:
  - Given corpus, infer structure implicit in the docs, without prior training.
- Supervised learning:
  - Train system to recognize docs of a certain type (e.g., docs in Italian, or docs about religion)
  - Decide whether or not new docs belong to the class(es) trained on

# Resources

- Good demo of results-list clustering: *cluster.cs.yale.edu*