CS347

Lecture 4

April 18, 2001

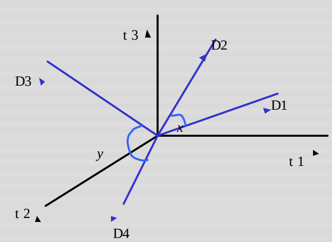
Today's topics

- Computing cosine-based ranking
- Speeding up cosine ranking
  - reducing the number of cosine computations
    - Union of term-wise candidates
    - Sampling and pre-grouping
  - reducing the number of dimensions
    - Random projection
    - Latent semantic indexing

Recall doc as vector

- Each doc *j* is a vector of $tf \times idf$ values, one component for each term.
- Can normalize to unit length.
- So we have a vector space
  - terms are axes
  - docs live in this space
  - even with stemming, may have 10000+ dimensions

Intuition



Postulate: Documents that are "close together" in vector space talk about the same things.

Cosine similarity

Cosine similarity of D_j, D_k:

$$sim(D_j, D_k) = \sum_{i=1}^{m} w_{ij} \times w_{ik}$$

Aka normalized inner product.

Can also compute cosine similarity from a query (vector of terms, e.g., ***truth forever***) to each document.

Exercises

- How would you augment the inverted index built in lectures 1-3 to support cosine ranking computations?
- Walk through the steps of serving a query.

Why use vector spaces?

- **Key**: A user's query can be viewed as a (very) short document.
- Query becomes a vector in the same space as the docs.
- Can measure each doc's cosine proximity to query → ranking.

Efficient cosine ranking

- Ranking consists of computing the *k* docs in the corpus "nearest" to the query ⇒ *k* largest query-doc cosines.
- Efficient ranking:
  - Computing a single cosine efficiently.
  - Choosing the *k* largest cosine values efficiently.

Computing a single cosine

- For every term i, with each doc j, store term frequency tf_{ij}.
  - Tradeoffs on whether to store term count, freq, or weighted by idf_i. (Coding possibilities.)
- Accumulate component-wise sum

$$sim(D_j, D_k) = \sum_{i=1}^{m} w_{ij} \times w_{ik}$$

More on speeding up a single cosine, later in this lecture.

Computing the *k* largest cosines: selection vs. sorting

- Typically we want to retrieve the top k docs (in the cosine ranking for the query)
  - not totally order all docs in the corpus
  - just pick off docs with k highest cosines.

Use heap for selecting top *k*

- Binary tree in which each node's value > values of children
- Takes $2n$ operations to construct, then each of $k \log n$ "winners" read off in $2 \log n$ steps.
- For n=1M, $k=100$, this is about 10% of the cost of sorting.

Bottleneck

- Still need to first compute cosines from query to each of n docs → several seconds for $n=1M$.
- Can select from only non-zero cosines; should be $<< 1M$.

Can we avoid this?

- Yes, but may occasionally get an answer wrong
  - a doc *not* in the top *k* may creep into the answer.

Term-wise candidates

- Preprocess: Pre-compute, for each term, its *k* nearest docs.
  - (Treat each term as a 1-term query.)
  - lots of preprocessing.
  - Result: “preferred list” for each term.
- Search:
  - For a *t*-term query, take the union of their *t* preferred lists - call this set *S*.
  - Compute cosines from the query to only the docs in *S*, and choose top *k*.

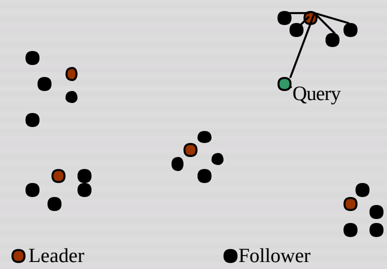
Exercises

- Fill in the details of the calculation:
  - Which docs go into the preferred list for a term?
- Devise a small example where this method gives an incorrect ranking.

Sampling and pre-grouping

- First run a pre-processing phase:
  - pick $\sqrt{n}$ *docs* at random: call these *leaders*
  - For each other doc, pre-compute nearest leader
    - Docs attached to a leader: its *followers;*
    - Likely: each leader has ~ $\sqrt{n}$ followers.
- Process a query as follows:
  - Given query *Q*, find its nearest *leader L.*
  - Seek *k* nearest docs from among *L*’s followers.

Visualization



Why use random sampling

- Fast
- Leaders reflect data distribution

General variants

- Have each follower attached to a=3 (say) nearest leaders.
- From query, find b=4 (say) nearest leaders and their followers.
- Can recur on leader/follower construction.

Exercises

- To find the nearest leader in step 1, how many cosine computations do we do?
- What is the effect of the constants a,b on the previous slide?
- Devise an example where this is *likely to* fail - we miss one of the k nearest docs.
  - *Likely* under random sampling.

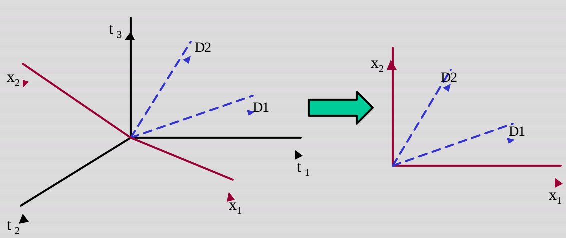
Dimensionality reduction

- What if we could take our vectors and “pack” them into fewer dimensions (say 10000→100) while preserving distances?
- (Well, almost.)
  - Speeds up cosine computations.
- Two methods:
  - Random projection.
  - “Latent semantic indexing”.

Random projection onto $k<<m$ axes.

- Choose a random direction x_1 in the vector space.
- For $i = 2$ to k,
  - Choose a random direction x_i that is orthogonal to $x_1, x_2, \ldots x_{i-1}$.
- Project each doc vector into the subspace $x_1, x_2, \ldots x_k$.

E.g., from 3 to 2 dimensions



x_1 is a random direction in (t_1,t_2,t_3) space.
x_2 is chosen randomly but orthogonal to x_1.

Guarantee

- With high probability, relative distances are (approximately) preserved by projection.
- Pointer to precise theorem in Resources.

Computing the random projection

- Projecting n vectors from m dimensions down to k dimensions:
  - Start with $m \times n$ matrix of terms $\times$ docs, A.
  - Find random $k \times m$ orthogonal projection matrix R.
  - Compute matrix product $W = R \times A$.
- jth column of W is the vector corresponding to doc j, but now in $k << m$ dimensions.

Cost of computation

- This takes a total of *kmn* multiplications. (Why?)
- Expensive - see Resources for ways to do essentially the same thing, quicker.
- *Exercise: by projecting from 10000 dimensions down to 100, are we really going to make each cosine computation faster?*

Latent semantic indexing (LSI)

- Another technique for dimension reduction
- Random projection was data-*independent*
- LSI on the other hand is data-*dependent*
  - Eliminate redundant axes
  - Pull together "related" axes
    - ***car*** and ***automobile***

Notions from linear algebra

- Matrix, vector
- Matrix transpose and product
- Rank
- Eigenvalues and eigenvectors.

Overview of LSI

- Pre-process docs using a technique from linear algebra called Singular Value Decomposition.
- Have control over the granularity of this process:
  - create new vector space, details to follow.
- Queries handled in this new vector space.

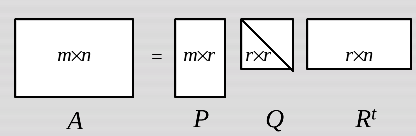
Singular-Value Decomposition

- Recall $m \times n$ matrix of terms $\times$ docs, A.
  - A has rank $r \leq m,n$.
- Define term-term correlation matrix $T=AA^t$
  - A^t denotes the matrix transpose of A.
  - T is a square, symmetric $m \times m$ matrix. ⇐ Why?
- Doc-doc correlation matrix $D=A^tA$.
  - D is a square, symmetric $n \times n$ matrix.

Eigenvectors

- Denote by P the $m \times r$ matrix of eigenvectors of T.
- Denote by R the $n \times r$ matrix of eigenvectors of D.
- It turns out A can be expressed (decomposed) as $A = PQR^t$
  - Q is a diagonal matrix with the eigenvalues of AA^t in sorted order.

Visualization

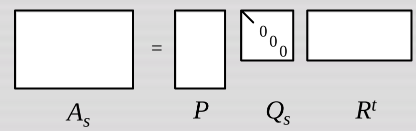


Dimension reduction

- For some $s << r$, zero out all but the s biggest eigenvalues in Q.
  - Denote by Q_s this new version of Q.
  - Typically s in the hundreds while r could be in the (tens of) thousands.
- Let $A_s = P\ Q_s\ R^t$
- Turns out A_s is a pretty good approximation to A.

We'll explain what this means.

Visualization



The columns of A_s represent the docs, but in $s<<m$ dimensions.

Guarantee

- Relative distances are (approximately) preserved by projection:
  - Of all $m \times n$ rank s matrices, A_s is the best approximation to A.
- Pointer to precise theorem in Resources.

Doc-doc similarities

- $A_s A_s^t$ is a matrix of doc-doc similarities:
  - the (j,k) entry is a measure of the similarity of doc j to doc k.

Semi-precise intuition

- We accomplish more than dimension reduction here:
  - Docs with lots of overlapping terms stay together
  - Terms from these docs also get pulled together.
- Thus ***car*** and ***automobile*** get pulled together because both co-occur in docs with ***tires, radiator, cylinder***, etc.

Query processing

- View a query as a (short) doc:
  - call it row 0 of A_s.
- Now the entries in row 0 of $A_s A_s^t$ give the similarities of the query with each doc.
- Entry *(0,j)* is the score of doc *j* on the query.
- *Exercise*: *fill in the details of scoring/ranking.*

Resources

- Random projection theorem: http://citeseer.nj.nec.com/dasgupta99elementary.html
- Faster random projection: http://citeseer.nj.nec.com/frieze98fast.html
- Latent semantic indexing: http://citeseer.nj.nec.com/deerwester90indexing.html
- Books: MG 4.6, MIR 2.7.2.