Low-Support, High-Correlation

Finding Rare but Similar Items

Minhashing

Locality-Sensitive Hashing

The Problem

- Rather than finding high-support item-pairs in basket data, look for items that are highly "correlated."
  - If one appears in a basket, there is a good chance that the other does.
  - "Yachts and caviar" as itemsets: low support, but often appear together.

Correlation Versus Support

- A-Priori and similar methods are useless for low-support, high-correlation itemsets.
- When support threshold is low, too many itemsets are frequent.
  - Memory requirements too high.
- A-Priori does not address correlation.

Matrix Representation of Item/Basket Data

- Columns = items.
- Rows = baskets.
- Entry (r, c) = 1 if item c is in basket r; = 0 if not.
- Assume matrix is almost all 0's.

In Matrix Form

	m	c	p	b	j
{m,c,b}	1	1	0	1	0
{m,p,b}	1	0	1	1	0
{m,b}	1	0	0	1	0
{c,j}	0	1	0	0	1
{m,p,j}	1	0	1	0	1
{m,c,b,j}	1	1	0	1	1
{c,b,j}	0	1	0	1	1
{c,b}	0	1	0	1	0

Applications --- (1)

- Rows = customers; columns = items.
  - (r, c) = 1 if and only if customer r bought item c.
  - Well correlated columns are items that tend to be bought by the same customers.
  - Used by on-line vendors to select items to "pitch" to individual customers.

Applications --- (2)

- Rows = (footprints of) shingles; columns = documents.
  - (r, c) = 1 iff footprint r is present in document c.
  - Find similar documents, as in Anand's 10/10 lecture.

Applications --- (3)

- Rows and columns are both Web pages.
  - (r, c) = 1 iff page r links to page c.
  - Correlated columns are pages with many of the same in-links.
  - These pages may be about the same topic.

Assumptions --- (1)

1. Number of items allows a small amount of main-memory/item.
   - E.g., main memory =

     Number of items * 100
2. Too many items to store anything in main-memory for each *pair* of items.

Assumptions --- (2)

3. Too many baskets to store anything in main memory for each basket.
4. Data is very sparse: it is rare for an item to be in a basket.

From Correlation to Similarity

- Statistical correlation is too hard to compute, and probably meaningless.
  - Most entries are 0, so correlation of columns is always high.
- Substitute "similarity," as in shingles-and-documents study.

Similarity of Columns

- Think of a column as the set of rows in which it has 1.
- The *similarity* of columns C_1 and C_2 = *Sim* (C_1,C_2) = is the ratio of the sizes of the intersection and union of C_1 and C_2.
  - *Sim* $(C_1,C_2) = |C_1 \cap C_2|/|C_1 \cup C_2|$ = *Jaccard measure*.

Example

C_1	C_2
0	1
1	0
1	1
0	0
1	1
0	1

$$\text{Sim}(C_1, C_2) = 2/5 = 0.4$$

Outline of Algorithm

1. Compute “signatures” (“sketches”) of columns = small summaries of columns.
   - Read from disk to main memory.
2. Examine signatures in main memory to find similar signatures.
   - Essential: similarity of signatures and columns are related.
3. Check that columns with similar signatures are really similar (optional).

Signatures

- Key idea: "hash" each column *C* to a small *signature* *Sig* (C), such that:
  1. *Sig* (C) is small enough that we can fit a signature in main memory for each column.
  2. *Sim* (C_1, C_2) is the same as the "similarity" of *Sig* (C_1) and *Sig* (C_2).

An Idea That Doesn't Work

- Pick 100 rows at random, and let the signature of column *C* be the 100 bits of *C* in those rows.
- Because the matrix is sparse, many columns would have 00. . .0 as a signature, yet be very dissimilar because their 1's are in different rows.

Four Types of Rows

- Given columns C_1 and C_2, rows may be classified as:

	C_1	C_2
a	1	1
b	1	0
c	0	1
d	0	0

- Also, *a* = # rows of type *a* , etc.
- Note $Sim(C_1, C_2) = a/(a+b+c)$.

Minhashing

- Imagine the rows permuted randomly.
- Define "hash" function $h(C)$ = the number of the first (in the permuted order) row in which column C has 1.
- Use several (100?) independent hash functions to create a signature.

Minhashing Example

Input matrix

1	4	3	1	0	1	0
3	2	4	1	0	0	1
7	1	7	0	1	0	1
6	3	6	0	1	0	1
2	6	1	0	1	0	1
5	7	2	1	0	1	0
4	5	5	1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Surprising Property

- The probability (over all permutations of the rows) that $h(C_1) = h(C_2)$ is the same as $Sim(C_1, C_2)$.
- Both are $a/(a+b+c)$!
- Why?
  - Look down columns C_1 and C_2 until we see a 1.
  - If it's a type a row, then $h(C_1) = h(C_2)$. If a type b or c row, then not.

Similarity for Signatures

- The similarity of signatures is the fraction of the rows in which they agree.
  - Remember, each row corresponds to a permutation or "hash function."

Min Hashing – Example

1	4	3
3	2	4
7	1	7
6	3	6
2	6	1
5	7	2
4	5	5

Input matrix

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col.-Col.	0.75	0.75	0	0
Sig.-Sig.	0.67	1.00	0	0

Minhash Signatures

- Pick (say) 100 random permutations of the rows.
- Think of *Sig* (C) as a column vector.
- Let *Sig* (C)[i] = row number of the first row with 1 in column C, for *i* th permutation.

Implementation --- (1)

- Number of rows = 1 billion.
- Hard to pick a random permutation from 1…billion.
- Representing a random permutation requires billion entries.
- Accessing rows in permuted order is tough!
  - The number of passes would be prohibitive.

Implementation --- (2)

1. Pick (say) 100 hash functions.
2. For each column c and each hash function h_i, keep a "slot" $M(i, c)$ for that minhash value.
3. **for** each row r, and for each column c with 1 in row r, and for each hash function h_i **do**

   if $h_i(r)$ is a smaller value than $M(i, c)$ **then**

   $M(i, c) := h_i(r)$.

- Needs only one pass through the data.

Example

Row	C1	C2
1	1	0
2	0	1
3	1	1
4	1	0
5	0	1

$h(x) = x \bmod 5$
$g(x) = 2x{+}1 \bmod 5$

$h(1) = 1$	1	-
$g(1) = 3$	3	-
$h(2) = 2$	1	2
$g(2) = 0$	3	0
$h(3) = 3$	1	2
$g(3) = 2$	2	0
$h(4) = 4$	1	2
$g(4) = 4$	2	0
$h(5) = 0$	1	0
$g(5) = 1$	2	0

Comparison with “Shingling”

- The shingling paper proposed using one hash function and taking the first 100 (say) values.
- Almost the same, but:
  - Faster --- saves on hash-computation.
  - Admits some correlation among rows of the signatures.

Candidate Generation

- Pick a similarity threshold s, a fraction < 1.
- A pair of columns c and d is a *candidate pair* if their signatures agree in at least fraction s of the rows.
  - I.e., $M(i, c) = M(i, d)$ for at least fraction s values of i.

The Problem with Checking Candidates

- While the signatures of all columns may fit in main memory, comparing the signatures of all pairs of columns is quadratic in the number of columns.
- Example: 10^6 columns implies $5*10^{11}$ comparisons.
- At 1 microsecond/comparison: 6 days.

Solutions

1. DCM method (Anand's 10/10 slides) relies on external sorting, so several passes over the data are needed.
2. *Locality-Sensitive Hashing* (LSH) is a method that can be carried out in main memory, but admits some false negatives.

Locality-Sensitive Hashing

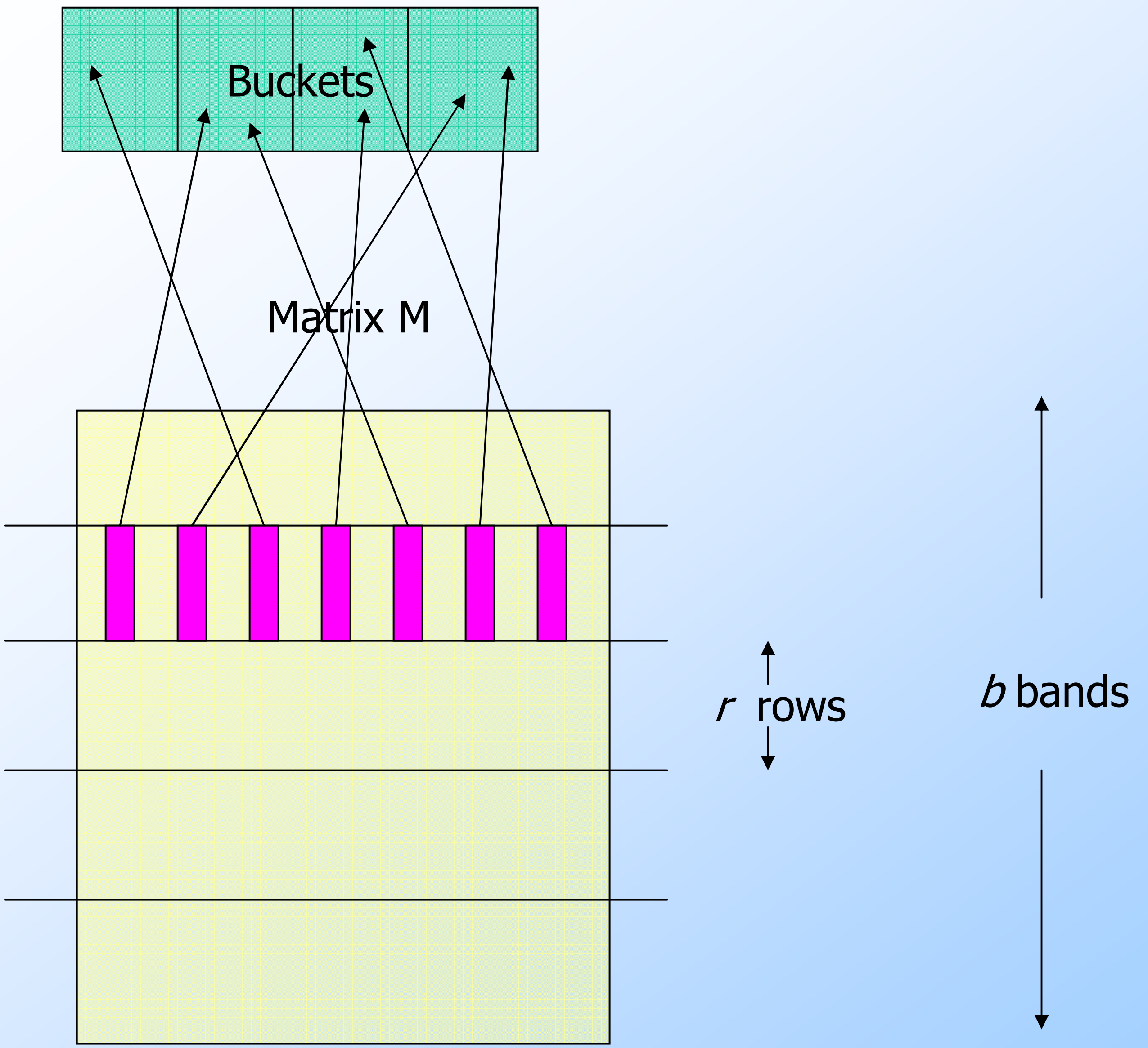
- Unrelated to "minhashing."
- Operates on signatures.
- Big idea: hash columns of signature matrix M several times.
- Arrange that similar columns are more likely to hash to the same bucket.
- Candidate pairs are those that hash at least once to the same bucket.

Partition into Bands

- Divide matrix M into b bands of r rows.
- For each band, hash its portion of each column to k buckets.
- *Candidate* column pairs are those that hash to the same bucket for ≥ 1 band.
- Tune b and r to catch most similar pairs, few nonsimilar pairs.

Simplifying Assumption

- There are enough buckets that columns are unlikely to hash to the same bucket unless they are identical in a particular band.
- Hereafter, we assume that “same bucket” means “identical.”


Buckets
Matrix M
r rows
b bands

Example

- Suppose 100,000 columns.
- Signatures of 100 integers.
- Therefore, signatures take 40Mb.
- But 5,000,000,000 pairs of signatures can take a while to compare.
- Choose 20 bands of 5 integers/band.

Suppose C_1, C_2 are 80% Similar

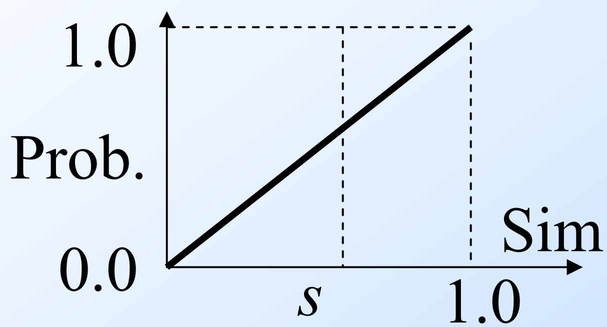
- Probability C_1, C_2 identical in one particular band: $(0.8)^5 = 0.328$.
- Probability C_1, C_2 are *not* similar in any of the 20 bands: $(1-0.328)^{20} = .00035$.
  - i.e., we miss about 1/3000th of the 80%-similar column pairs.

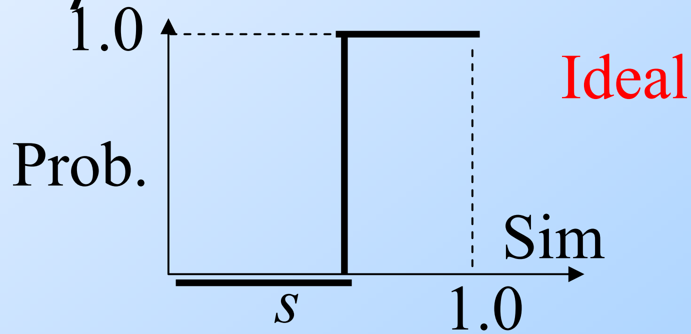
Suppose C_1, C_2 Only 40% Similar

- Probability C_1, C_2 identical in any one particular band: $(0.4)^5 = 0.01$.
- Probability C_1, C_2 identical in ≥ 1 of 20 bands: $\leq 20 * 0.01 = 0.2$.
- Small probability C_1, C_2 not identical in a band, but hash to the same bucket.
- But false positives much lower for similarities << 40%.

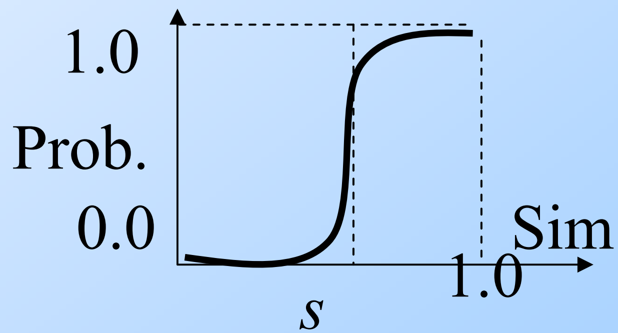
LSH --- Graphically

- Example Target: All pairs with *Sim* > 60%.
- Suppose we use only one hash function:





LSH (partition into bands) gives us:



$$1-(1-s^r)^b$$

LSH Summary

- Tune to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures.
- Check in main memory that candidate pairs really do have similar signatures.
- Then, in another pass through data, check that the remaining candidate pairs really are similar *columns* .

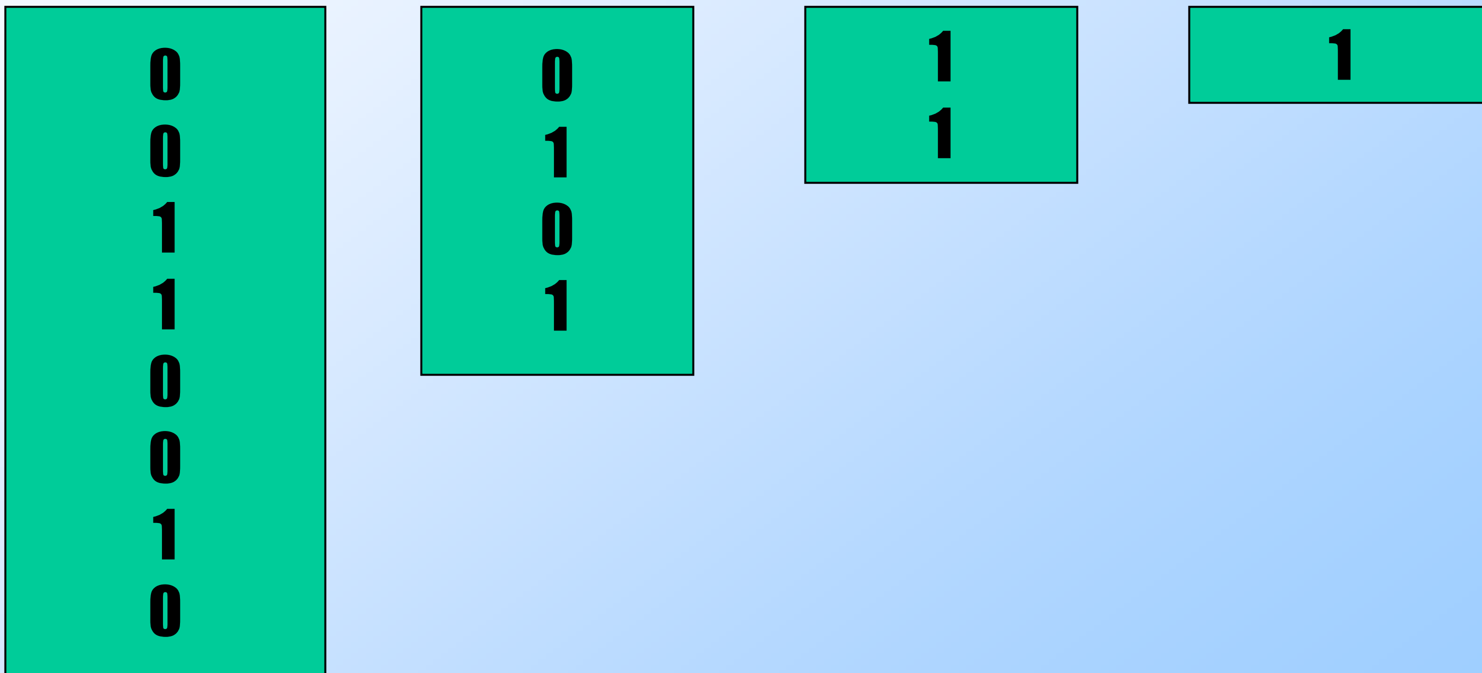
New Topic: Hamming LSH

- An alternative to minhash + LSH.
- Takes advantage of the fact that if columns are not sparse, random rows serve as a good signature.
- Trick: create data matrices of exponentially decreasing sizes, increasing densities.

Amplification of 1's

- *Hamming LSH* constructs a series of matrices, each with half as many rows, by OR-ing together pairs of rows.
- Candidate pairs from each matrix have (say) between 20% - 80% 1's and are similar in selected 100 rows.
  - 20%-80% OK for similarity thresholds ≥ 0.5. Otherwise, two "similar" columns could fail to both be in range for at least one matrix.

Example



Using Hamming LSH

- Construct the sequence of matrices.
  - If there are R rows, then $\log_2 R$ matrices.
  - Total work = twice that of reading the original matrix.
- Use standard LSH to identify similar columns in each matrix, but restricted to columns of “medium” density.