# Multivalued Dependencies

## Fourth Normal Form

## Reasoning About FD's + MVD's

# Definition of MVD

- A *multivalued dependency* (MVD) on *R*, *X* ->-> *Y*, says that if two tuples of *R* agree on all the attributes of *X*, then their components in *Y* may be swapped, and the result will be two tuples that are also in the relation.
- i.e., for each value of *X*, the values of *Y* are independent of the values of *R*-*X*-*Y*.

# Example: MVD

Drinkers(name, addr, phones, beersLiked)

- A drinker's phones are independent of the beers they like.
  - name->->phones and name ->->beersLiked.
- Thus, each of a drinker's phones appears with each of the beers they like in all combinations.
- This repetition is unlike FD redundancy.
  - name->addr is the only FD.

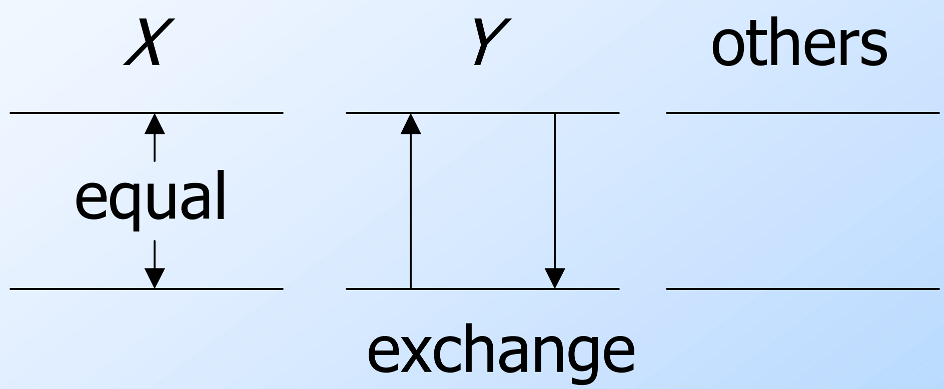
# Tuples Implied by name->->phones

If we have tuples:

| name | addr | phones | beersLiked |
|---|---|---|---|
| sue | a | p1 | b1 |
| sue | a | p2 | b2 |
| sue | a | p2 | b1 |
| sue | a | p1 | b2 |

Then these tuples must also be in the relation.

# Picture of MVD $X$ ->-> $Y$



# MVD Rules

- Every FD is an MVD (*promotion* ).
  - If *X* -> *Y*, then swapping *Y* 's between two tuples that agree on *X* doesn't change the tuples.
  - Therefore, the "new" tuples are surely in the relation, and we know *X* ->-> *Y*.
- *Complementation* : If *X* ->-> *Y*, and *Z* is all the other attributes, then *X* ->-> *Z*.

# Splitting Doesn't Hold

- Like FD's, we cannot generally split the left side of an MVD.
- But unlike FD's, we cannot split the right side either --- sometimes you have to leave several attributes on the right side.

# Example: Multiattribute Right Sides

Drinkers(name, areaCode, phone, beersLiked, manf)

- A drinker can have several phones, with the number divided between areaCode and phone (last 7 digits).
- A drinker can like several beers, each with its own manufacturer.

# Example Continued

- Since the areaCode-phone combinations for a drinker are independent of the beersLiked-manf combinations, we expect that the following MVD's hold:

    name ->-> areaCode phone

    name ->-> beersLiked manf

# Example Data

Here is possible data satisfying these MVD's:

| name | areaCode | phone | beersLiked | manf |
|---|---|---|---|---|
| Sue | 650 | 555-1111 | Bud | A.B. |
| Sue | 650 | 555-1111 | WickedAle | Pete's |
| Sue | 415 | 555-9999 | Bud | A.B. |
| Sue | 415 | 555-9999 | WickedAle | Pete's |

But we cannot swap area codes or phones by themselves. That is, neither name->->areaCode nor name->->phone holds for this relation.

# Fourth Normal Form

- The redundancy that comes from MVD's is not removable by putting the database schema in BCNF.
- There is a stronger normal form, called 4NF, that (intuitively) treats MVD's as FD's when it comes to decomposition, but not when determining keys of the relation.

# 4NF Definition

- A relation *R* is in *4NF* if: whenever *X* ->-> *Y* is a nontrivial MVD, then *X* is a superkey.
  - *Nontrivial MVD* means that:
    1. *Y* is not a subset of *X*, and
    2. *X* and *Y* are not, together, all the attributes.
  - Note that the definition of "superkey" still depends on FD's only.

# BCNF Versus 4NF

- Remember that every FD $X \text{->} Y$ is also an MVD, $X \text{->->} Y$.
- Thus, if $R$ is in 4NF, it is certainly in BCNF.
  - Because any BCNF violation is a 4NF violation (after conversion to an MVD).
- But $R$ could be in BCNF and not 4NF, because MVD's are "invisible" to BCNF.

# Decomposition and 4NF

- If $X$->->$Y$ is a 4NF violation for relation $R$, we can decompose $R$ using the same technique as for BCNF.
  1. $XY$ is one of the decomposed relations.
  2. All but $Y - X$ is the other.

# Example: 4NF Decomposition

Drinkers(name, addr, phones, beersLiked)

FD: name -> addr

MVD's: name ->-> phones

name ->-> beersLiked

- Key is {name, phones, beersLiked}.
- All dependencies violate 4NF.

# Example Continued

- Decompose using name -> addr:

1. Drinkers1(name, addr)
   - In 4NF; only dependency is name -> addr.
2. Drinkers2(name, phones, beersLiked)
   - Not in 4NF. MVD's name ->-> phones and name ->-> beersLiked apply. No FD's, so all three attributes form the key.

# Example: Decompose Drinkers2

- Either MVD name ->>-> phones or name ->>-> beersLiked tells us to decompose to:
  - Drinkers3(name, phones)
  - Drinkers4(name, beersLiked)

# Reasoning About MVD's + FD's

- Problem: given a set of MVD's and/or FD's that hold for a relation *R*, does a certain FD or MVD also hold in *R* ?
- Solution: Use a tableau to explore all inferences from the given set, to see if you can prove the target dependency.

# Why Do We Care?

1. 4NF technically requires an MVD violation.
   - Need to infer MVD’s from given FD’s and MVD’s that may not be violations themselves.
2. When we decompose, we need to project FD’s + MVD’s.

# Example: Chasing a Tableau With MVD's and FD's

- To apply a FD, equate symbols, as before.
- To apply an MVD, generate one or both of the tuples we know must also be in the relation represented by the tableau.
- We'll prove: if A->->BC and D->C, then A->C.

# The Tableau for A->C

Goal: prove that $c_1 = c_2$.

| $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|
| $a$ | $b_1$ | ~~$c_1$~~ $c_2$ | $d_1$ |
| $a$ | $b_2$ | $c_2$ | $d_2$ |
| $a$ | $b_2$ | $c_2$ | $d_1$ |

Use A->->BC (first row's $D$ with second row's $BC$).

Use D->C (first and third row agree on $D$, therefore agree on $C$).

# Example: Transitive Law for MVD's

- If A->->B and B->->C, then A->->C.
  - Obvious from the complementation rule if the Schema is *ABC*.
  - But it holds no matter what the schema; we'll assume *ABCD*.

# The Tableau for A->->C

Goal: derive tuple $(a, b_1, c_2, d_1)$.

| $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|
| $a$ | $b_1$ | $c_1$ | $d_1$ |
| $a$ | $b_2$ | $c_2$ | $d_2$ |
| $a$ | $b_1$ | $c_2$ | $d_2$ |
| $a$ | $b_1$ | $c_2$ | $d_1$ |

Use A->->B to swap *B* from the first row into the second.

Use B->->C to swap *C* from the third row into the first.

# Rules for Inferring MVD's + FD's

- Start with a tableau of two rows.
  - These rows agree on the attributes of the left side of the dependency to be inferred.
  - And they disagree on all other attributes.
  - Use unsubscripted variables where they agree, subscripts where they disagree.

# Inference: Applying a FD

- Apply a FD X->Y by finding rows that agree on all attributes of *X*. Force the rows to agree on all attributes of *Y*.
  - Replace one variable by the other.
  - If the replaced variable is part of the goal tuple, replace it there too.

# Inference: Applying a MVD

- Apply a MVD X->->Y by finding two rows that agree in *X*.
  - Add to the tableau one or both rows that are formed by swapping the *Y*-components of these two rows.

# Inference: Goals

- To test whether U->V holds, we succeed by inferring that the two variables in each column of V are actually the same.
- If we are testing U->>->V, we succeed if we infer in the tableau a row that is the original two rows with the components of V swapped.

# Inference: Endgame

- Apply all the given FD's and MVD's until we cannot change the tableau.
- If we meet the goal, then the dependency is inferred.
- If not, then the final tableau is a counterexample relation.
  - Satisfies all given dependencies.
  - Original two rows violate target dependency.