CS109B Notes for Lecture 4/26/95

From RE's to Automata

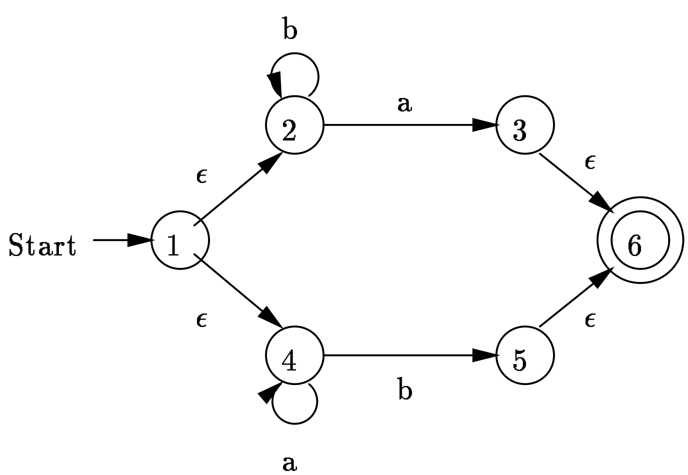
1. NFA's with ϵ-transitions. (ϵ-NFA's).
2. RE's $\rightarrow$ ϵ-NFA's.
3. ϵ-NFA's $\rightarrow$ NFA's.

ϵ-NFA's

Allow transition on ϵ.

- ϵ is invisible as far as the string labeling the part from start state to accepting state is concerned.

Example: $a^*b \mid b^*a$ is accepted by the following ϵ-NFA.



RE to ϵ-NFA

Produce a special kind of ϵ-NFA:

- One start, one accepting state.
- At most 2 arcs out of any state.

Construction of ϵ-NFA from RE is a structural induction on the expression tree for the RE.

- See pp. 574–5, FCS for pictures.

Basis: Operand: $\emptyset$, ϵ, or a symbol a.

Induction: Cases for |, concatenation, *.

- Inductive hypothesis $S(R)$: the ϵ-NFA constructed for RE R has paths from start to accepting state labeled by all and only the strings in $L(R)$.

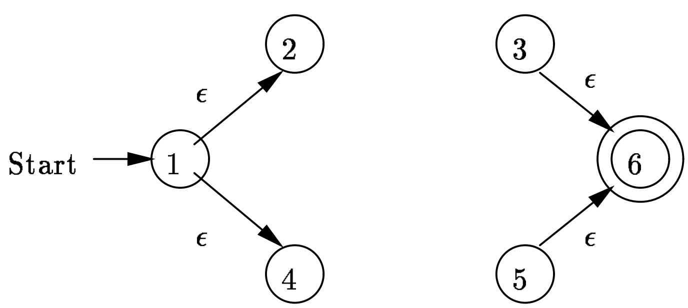
ϵ-NFA to NFA

First step is to determine for all states s and t whether there is a path labeled ϵ from s to t.

- Special case of all-pairs shortest path: give ϵ-arc a weight 0 and other arcs or no arc a weight ∞.

  - Ask: is the distance from s to t 0?

Example: Here is the above ϵ-NFA with non-ϵ arcs removed.



Here are the reaching pairs:

	1	2	3	4	5	6
1	1	1	0	1	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	1
4	0	0	0	1	0	0
5	0	0	0	0	1	1
6	0	0	0	0	0	1

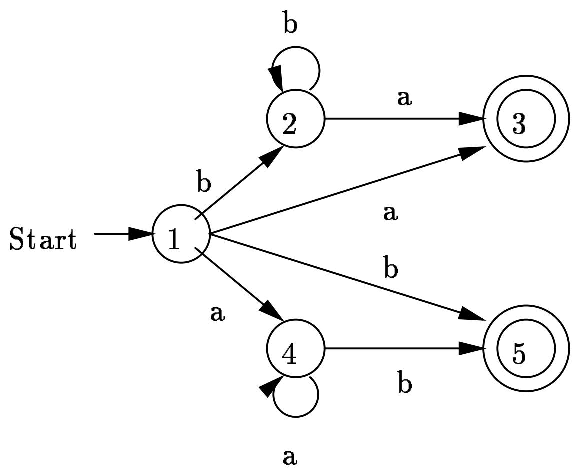
- *Important state* = start state or a state with a non-ϵ transition in.

Example: For our running example, all but 6 are important.

- Eliminate ϵ-transitions by:

- ☐ If there is an ϵ-path from important state s to t and a transition on t to r on symbol a (therefore r is surely important), then add a transition from s to r on a.
- ☐ Important state s is accepting iff there is a (possibly empty) ϵ-path from s to an accepting state.

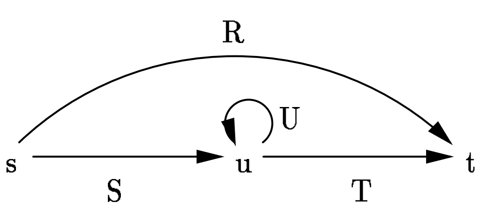
Example:

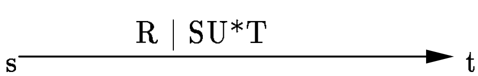


FA to RE

Key idea: *pivot* on a state (like Floyd's algorithm).

- Picture, p. 583, FCS.
- Initially, label of a FA arc is treated as a RE.
- If we pivot on state u, consider a predecessor state s and a successor state t.





- New RE for going from s to t is $R \mid SU^*T$. Why?

Reducing the Automaton

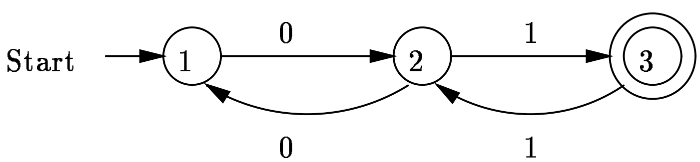
If there is one accepting state, and it is not the start state, eliminate all other states.

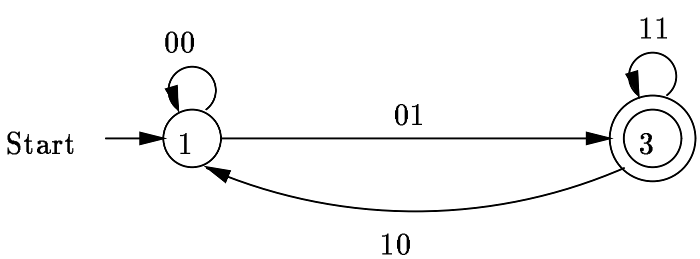
- The result is a 2-state automaton with RE's on 4 arcs. Fig. 10.43, p. 586,FCS, gives the automaton and the resulting RE.

Some additional details:

- If start = accepting, you get a 1-state automaton as in Fig. 10.44.
- If there is more than 1 accepting state, repeat process for each and take the union of the resulting RE's.

Example:





Resulting RE: $(00)^*01\Big(11 \mid 10(00)^*01\Big)^*$.